Towards Adjoint-Based Assimilation of SWOT Data in a Global Ocean Circulation Model

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Research Overview

The Surface Water and Ocean Topography (SWOT) mission provides unprecedented spatial resolution, resolving fine-scale ocean features. However, its 21-day repeat cycle leaves substantial spatiotemporal gaps, limiting the observation of rapidly evolving processes. Therefore, data assimilation is essential to reconstruct the evolving ocean state and to improve understanding of ocean circulation and energy transfer across scales.

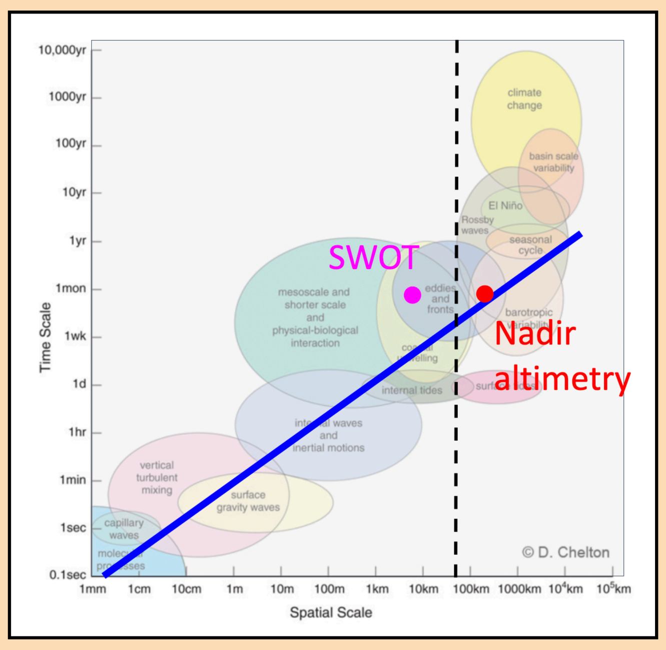


Figure 1. SWOT does not have sufficient sampling frequency to fully describe mesoscale eddies and internal tides. *Courtesy of Sarah Gille, adapted from D. Chelton*

GOAL: Develop a **global**, $\frac{1}{12}^{\circ}$ **resolution**, eddy- and tide-admitting **adjoint-based** state estimate constrained by SWOT observations.

CHALLENGE: Traditional adjoint methods are infeasible at high resolutions due to strong nonlinearities and large computational costs.

APPROACH: Implement a multi-grid adjoint using automatic differentiation that approximates the adjoint model on a coarser grid.

Ocean State Estimation

GLOBAL DOMAIN

A global ocean state-estimate will match the near-global observational coverage of SWOT, as well as that of nadir altimetry, which we also include in our assimilation.

$\frac{1}{12}^{\circ}$ RESOLUTION

Such a high-resolution discretization of a global general circulation model is required to permit mesoscale eddies and internal tides, which are critical for capturing the small-scale dynamics observed by SWOT.

ADJOINT-BASED DATA ASSIMILATION

Unlike sequential methods, adjoint-based data assimilation approaches enforce dynamical consistency and are fully interpretable. They act as dynamically and kinematically consistent interpolators to fill in observational gaps and connect surface topography to the ocean interior.

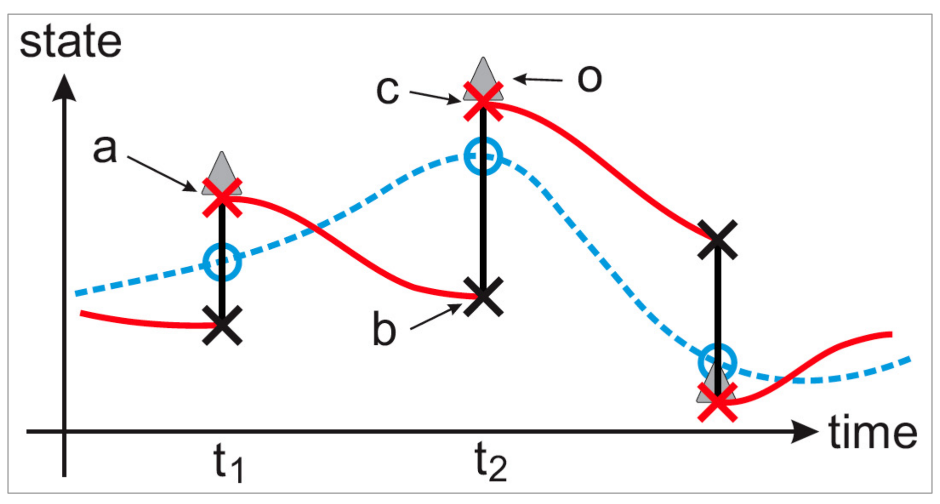


Figure 2. The analysis increment of sequential DA violates the equations of motion, while adjoint-based DA maintains dynamical consistency.

References

[1] M. Janiskov´ a and P. Lopez, *Linearized physics for data assimilation at ecmwf*, in Data Assimilation for Atmospheric, Oceanic and Hydrological Applications (Vol. II), S. K. Park and L. Xu, eds., Springer-Verlag Berlin Heidelberg, 2013, pp. 251–286

[2] R. Morrow, L.-L. Fu, F. Ardhuin, et al., *Global observations of fine-scale ocean surface topography with the surface water and ocean topography (swot) mission*, Frontiers in Marine Science, 6 (2019)

Multi-Grid Adjoint Approach

TRADITIONAL ADJOINT METHODS

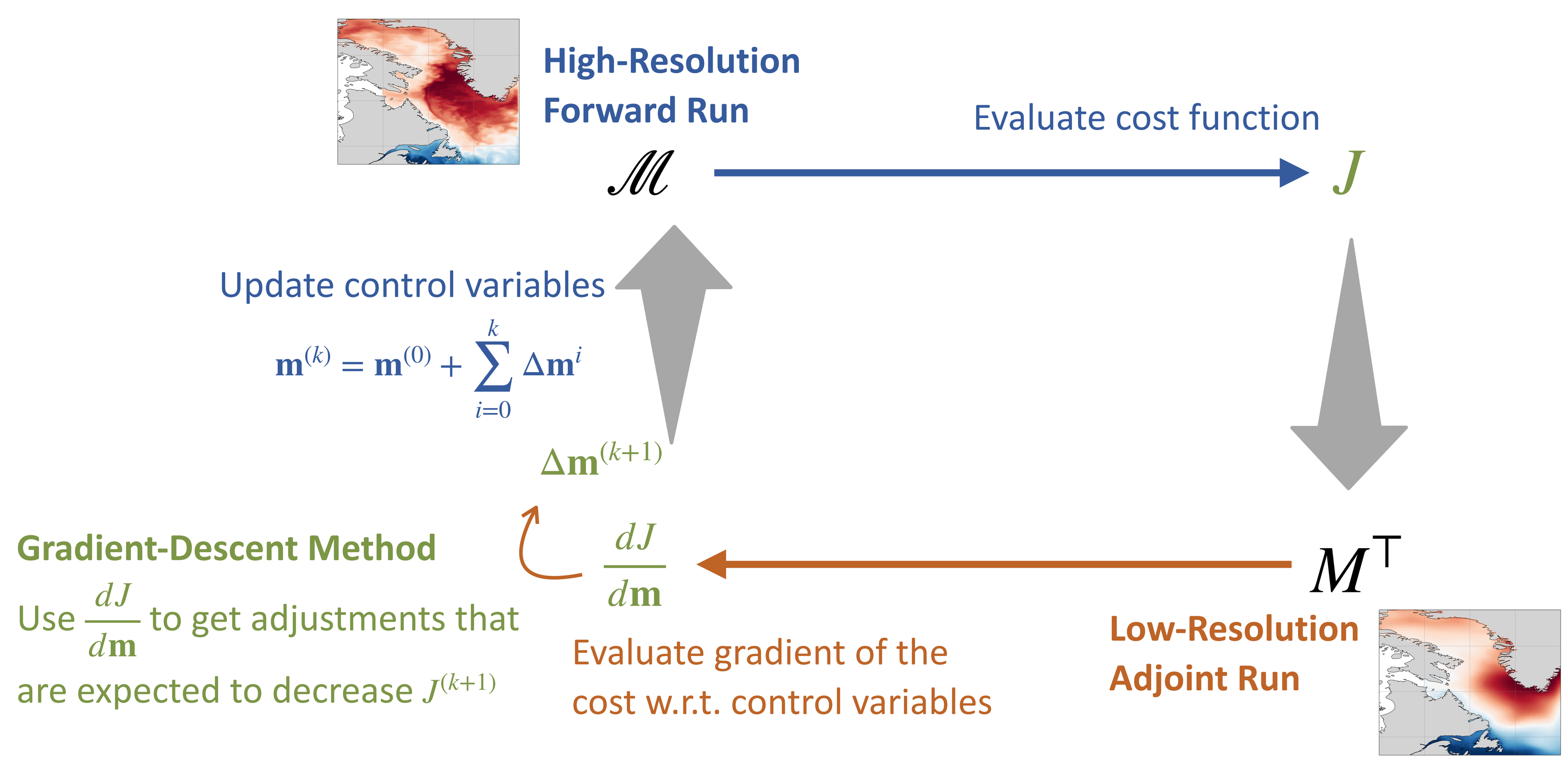
The adjoint-based state estimation framework brings the model state into agreement with observations, within error estimates, by adjusting a set of control variables $\mathbf{m}$ (such as initial conditions or surface boundary conditions). This approach is formulated as an optimization problem, where a cost function J is minimized with respect to $\mathbf{m}$, subject to the governing model equations.

$$J(\mathbf{m}) := \sum_{t=t_0}^{T} \frac{(\text{model}(\mathbf{m})_t - \text{obs}_t)^2}{\sigma_t} + \text{regularization}$$

Adjoint methods provide a way to compute the gradient. This allows us to compute the sensitivity of J to *all* control variables with just one forward/adjoint run, rather than *many* forward perturbation runs.

MULTI-GRID ADJOINT METHODS

Approximating the adjoint model on a coarser grid **improves stability and significantly reduces computational cost.**



Multi-Grid Adjoint Method in Use

As a preliminary test, we implement the multi-grid methodology in the Labrador Sea, a small regional domain, with a $\frac{1}{12}^{\circ}$ resolution forward model and $\frac{1}{3}^{\circ}$ resolution adjoint model.

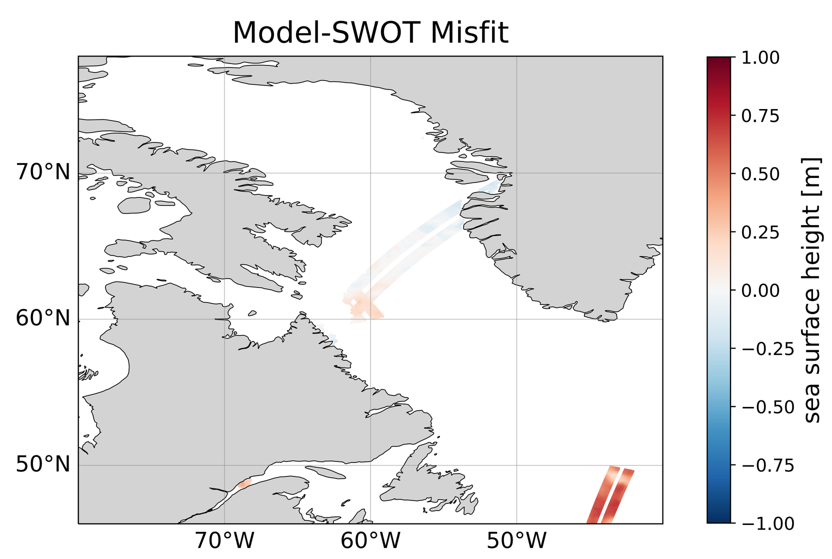


Figure 3. The misfit, used in the cost calculation, compares model output to SWOT data in January 2024.

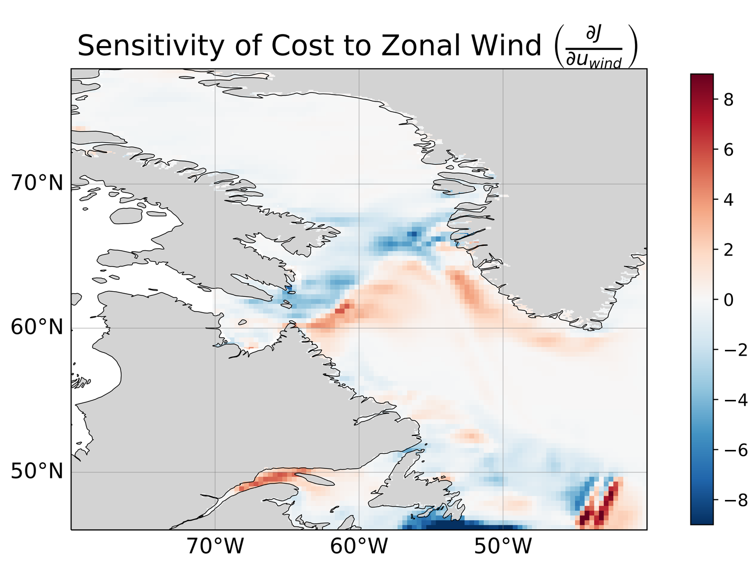


Figure 4. Sensitivities of the model-data misfit to atmospheric forcings are strongest near SWOT swaths.

- ➡ The memory requirement decreases from 14.5 GB to < 1 GB.
- ➡ We obtain a **stable** approximation to the high-resolution adjoint that is otherwise computationally intractable.

Future Work

1. Create a **state estimate based on SWOT** and nadir altimetry observations in a $\frac{1}{12}^{\circ}$ resolution configuration of the Labrador Sea.
2. Extend the regional state estimate to a **global ocean** state estimate.

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