Spectral Kinetic Energy Cascades from SWOT Observations in the Kuroshio Region

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Introduction / Motivation

Understanding how energy cascades across scales is key to linking mesoscale eddies, internal tides, and small-scale balanced motions. The new Surface Water and Ocean Topography (SWOT) mission provides unprecedented sea surface height (SSH) detail at ~15–30 km scales, but methods to infer spectral energy fluxes from SWOT data remain challenging. Here, we benchmark flux estimates against a high-resolution LLC4320 ocean simulation in the Kuroshio region as a step toward using SWOT for cross-scale energy diagnostics.

Data

SWOT: 21-day Level-3 Low-Rate SSH from passes 129, 157, 185, 213 over the Kuroshio region (140-152°E, 29-41°N).

LLC4320: 1/48° MITgcm simulation (η, U, V) spanning the nearby region (150-185°E, 25-45°N) and time for validation and method development.

Methods

1. Geostrophic velocity: Computed from SSH using finite differences with latitude-dependent f,

$$\psi = \frac{g \times \text{geostrophic SSH}}{f}$$ (Scott & Wang 2005)

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

2. Vorticity & Jacobian: SSH-derived velocity fields to evaluate nonlinear energy transfer.

$$\zeta = \nabla^2 \psi\,, J(\psi, \zeta) = \psi_x \zeta_y - \psi_y \zeta_x$$

3. Spectra: Wavenumber and frequency SSH spectra computed with Tukey windowing.

4. Spectral flux: Two alternative nonlinear energy transfer formulations, assumed equivalent under 2-D geostrophic flow:

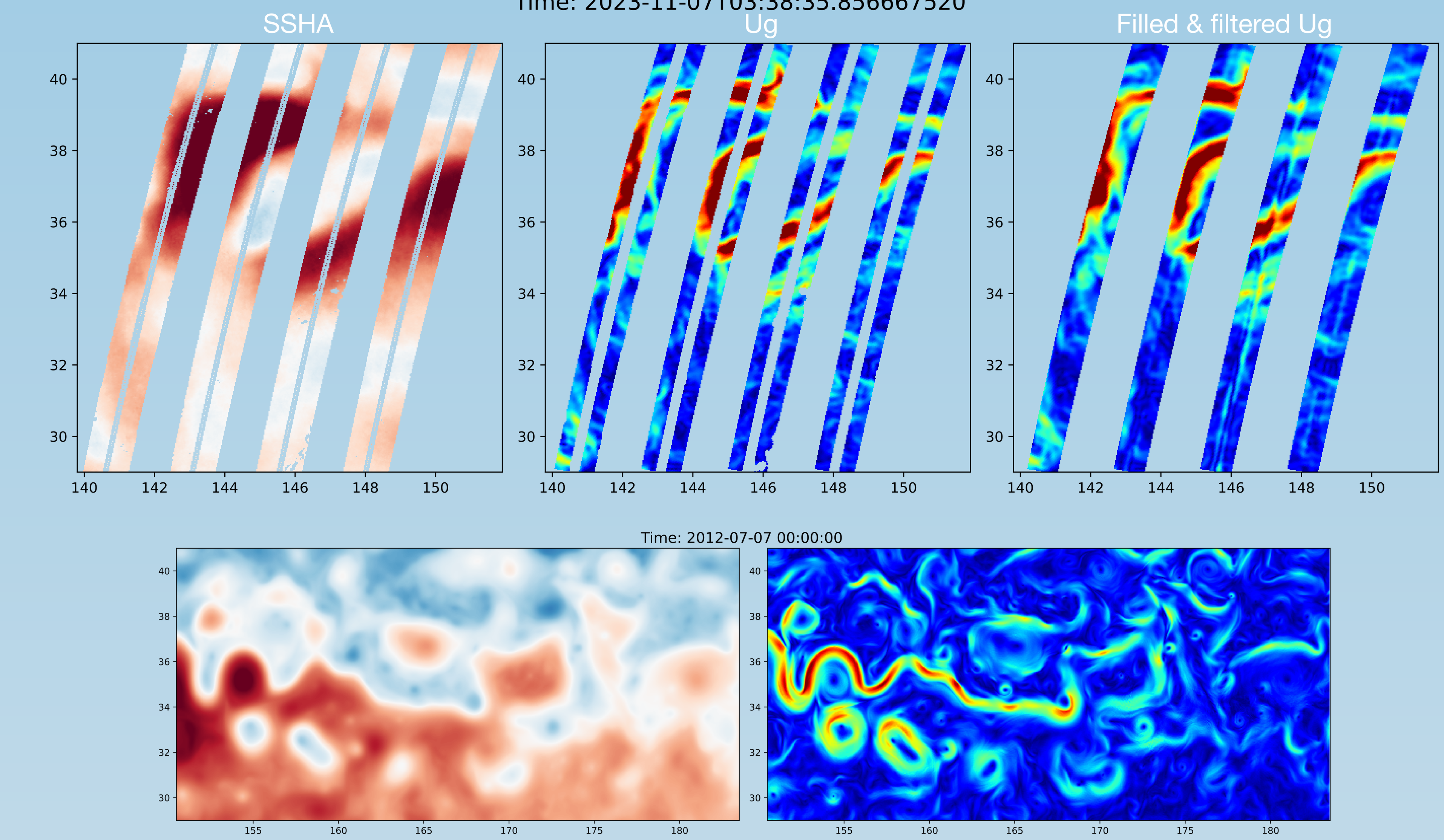
$$T(\mathbf{k}) = \mathrm{Re}\left[\hat{\psi}^*(\mathbf{k})\,\widehat{J(\psi,\zeta)}(\mathbf{k})\right]$$

$$T(\mathbf{k}) = \mathrm{Re}\left[\hat{\mathbf{u}}^*(\mathbf{k})\,\widehat{(\mathbf{u}\cdot\nabla)\mathbf{u}}(\mathbf{k})\right]$$

Cumulative flux is $\Pi(k) = -\int_k^\infty T(k')\,\mathrm{d}k'$

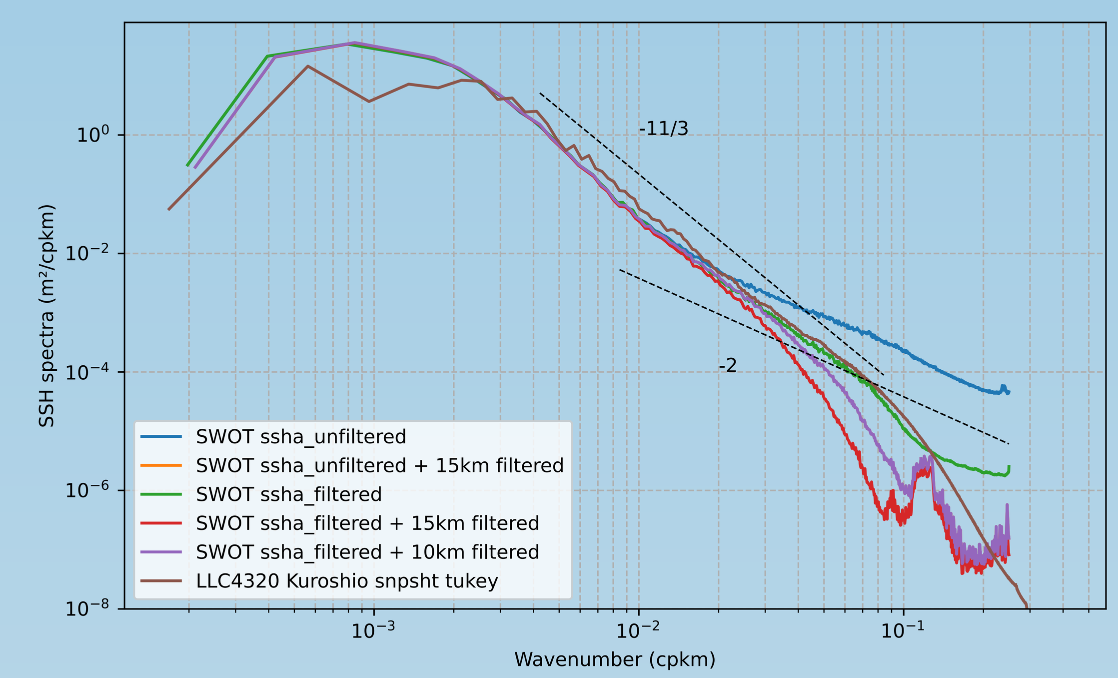
5. Computation: Fluxes are calculated as
   - 2-D isotropic flux: Derived from LLC4320 fields and radially averaged in wavenumber space.
   - 1-D flux: Computed along individual longitude (or along-track) slices from LLC4320 and SWOT, then averaged over time and longitude.

SWOT 21-day SSH and geostrophic velocity



SWOT swaths (140–152°E, 29–41°N) and LLC4320 model fields (150–185°E, 25–45°N) partially overlap in the Kuroshio region. Rd-Bu shading shows SSH anomaly; and jet shows geostrophic speed.

SSH Spectra

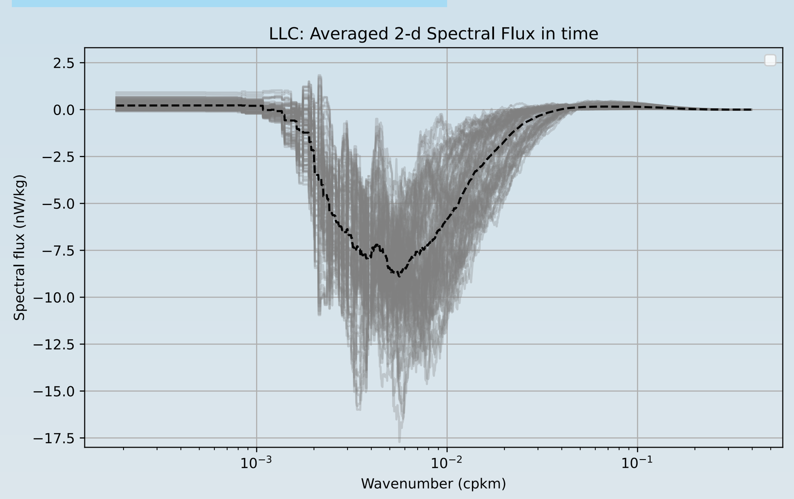


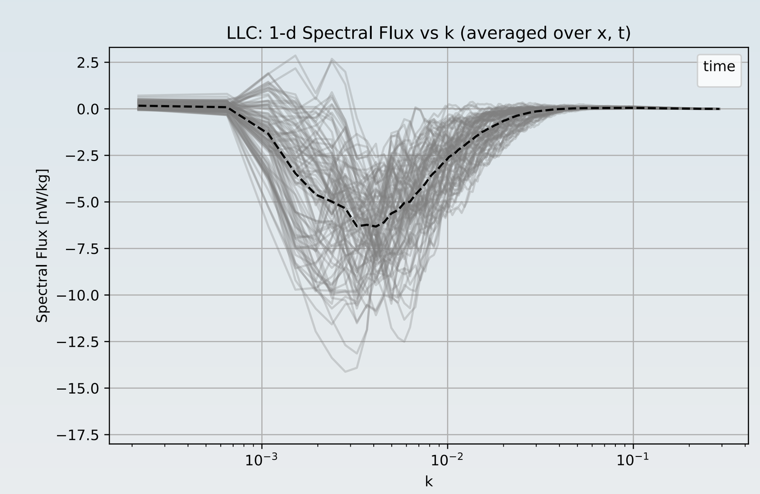
- SSH wavenumber spectra for SWOT and LLC4320 over the Kuroshio region show consistent spectral slopes.
- The official SWOT filtered SSH reduces high-wavenumber energy starting near k≈2×10^{-2} cpkm, yielding a slope close to−11/3 up to k>10^{-1} cpkm.
- Applying additional Gaussian filters (10–15 km) to the unfiltered or filtered SSH produces further energy reduction.
- The LLC4320 hourly η field closely matches the filtered SWOT spectra, both exhibiting an approximately −11/3 slope over the mesoscale–submesoscale range.

Spectral Flux

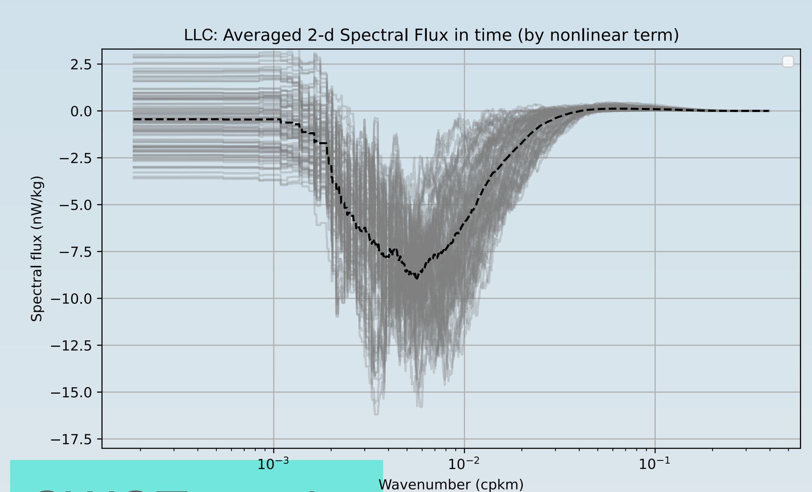
Spectral energy fluxes (Π(k)) were computed using both the Jacobian and nonlinear-velocity formulations.
Here, positive Π(k) indicates a forward (downscale) energy cascade, and negative Π(k) denotes an inverse (upscale) transfer.

LLC4320 results

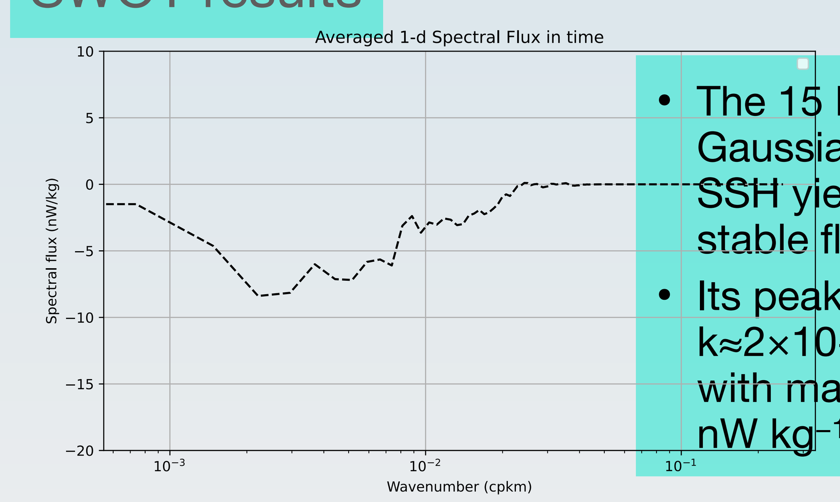




- All cases show dominant negative Π(k), signifying a inverse cascade with no forward cascade at high k.
- The corresponding 1-D along-track flux, averaged in longitude and time, peaks near k≈4×10^{-3} cpkm with amplitudes around+6 nW kg^{-1}, slightly weaker and shifted from the 2-D peak (k≈5×10^{-3} cpkm, +8 nW kg^{-1}).
- Fluxes obtained from the nonlinear-velocity formulation show nearly identical mean structures; individual snapshots can display small residuals at low k that average out over time.



SWOT results



- The 15 km Gaussian-filtered SSH yields the most stable flux estimate.
- Its peak occurs near k≈$2\times10{-3}$ cpkm with magnitude +8 nW kg^{-1}.

Discussion & Future work

The relationship between the 1-D and 2-D spectral flux formulations will be further analyzed in the context of their physical equivalence and complementarity.

The mathematical linkage between along-track (1-D) and isotropic (2-D) fluxes has been recently derived in Appendix of Qiu & Chen (2025, J. Phys. Oceanogr).

We will examine how these formulations compare in representing energy transfers in the Kuroshio region using SWOT observations.

Future efforts will investigate how different spatial filters affect the computed spectral fluxes and the associated cascade scales.

We also plan to apply wave–flow decomposition methods to separate balanced and unbalanced motions before computing Π(k).

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Reference:

Scott, R. B., and F. Wang, 2005: Direct evidence of an oceanic inverse kinetic energy cascade from satellite altimetry. J. Phys. Oceanogr., 35(9), 1650–1666. https://doi.org/10.1175/JPO2771.1

Qiu, B., and S. Chen, 2025: Fine-Scale Upper-Ocean Variability in the Kuroshio Extension Region from the Wide-Swath SWOT Measurements. J. Phys. Oceanogr. https://doi.org/10.1175/JPO-D-25-0042.1