Processing SWOT data with image restoration techniques

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Motivation: prepare the assimilation of SWOT in ocean models

The assimilation/inversion problem more or less consists in minimizing

$$\mathcal{J}(\mathbf{x}) = \mathrm{SomeNorm}(\mathbf{x} - \mathbf{x}^{\mathrm{prior}}) + [\mathbf{y} - \mathcal{H}(\mathbf{x})]^T \, \mathbf{R}^{-1} \, [\mathbf{y} - \mathcal{H}(\mathbf{x})]$$

solution of the inversion

First guess (a model output)

SWOT data

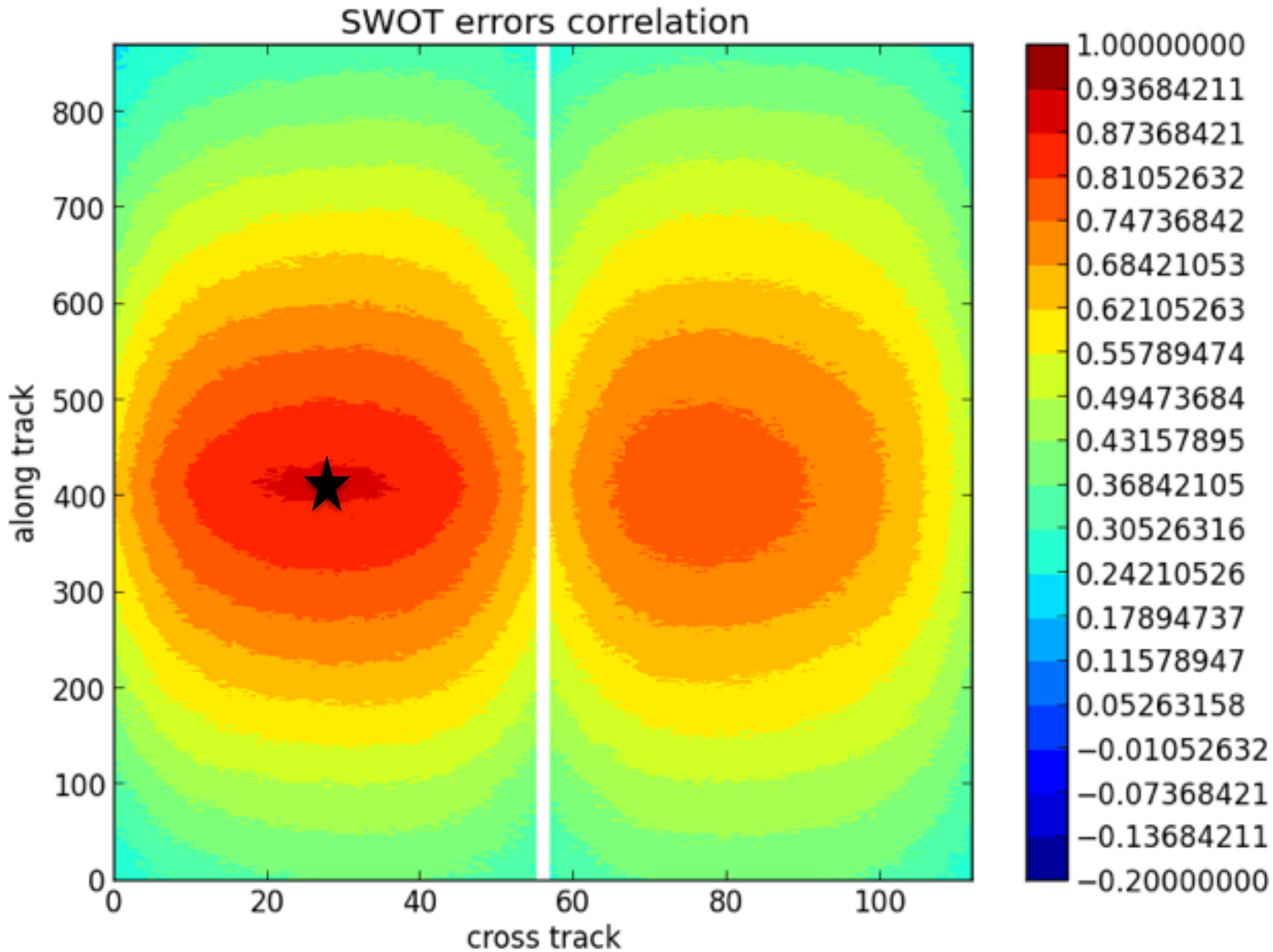
SWOT error covariance matrix

- The inverse of the error covariance matrix is required;
- Most assimilation algorithms are designed for diagonal matrices.

Spatial correlations in SWOT errors

(computed from 5000 realizations of the SWOT ocean simulator)

★ Reference point



Implication for the assimilation of SWOT data

$$\mathcal{J}(\mathbf{x}) = \mathrm{SomeNorm}(\mathbf{x} - \mathbf{x}^{\mathrm{prior}}) + [\mathbf{y} - \mathcal{H}(\mathbf{x})]^T \, \mathbf{R}^{-1} \, [\mathbf{y} - \mathcal{H}(\mathbf{x})]$$

solution of the inversion

First guess (a model output)

SWOT data

SWOT error covariance matrix

- The inverse of the covariance matrix is required;
- Most assimilation algorithms are designed for diagonal matrices.
- With SWOT, this matrix is big and non diagonal.

Trick: transform the observation vector (Brankart et al, 2009)

We introduce

$$\mathbf{T} = \begin{pmatrix} \mathbf{I} \\ \delta_a \\ \delta_c \\ \delta_a^2 \\ \delta_c^2 \end{pmatrix}$$

— Identity

— Along-track and across-track derivatives

— Second-order derivatives

$$\mathbf{y}^+ = \mathbf{T}\mathbf{y} \qquad \mathcal{H}^+ = \mathbf{T}\mathcal{H}$$

And we find the appropriate diagonal R^+ so that:

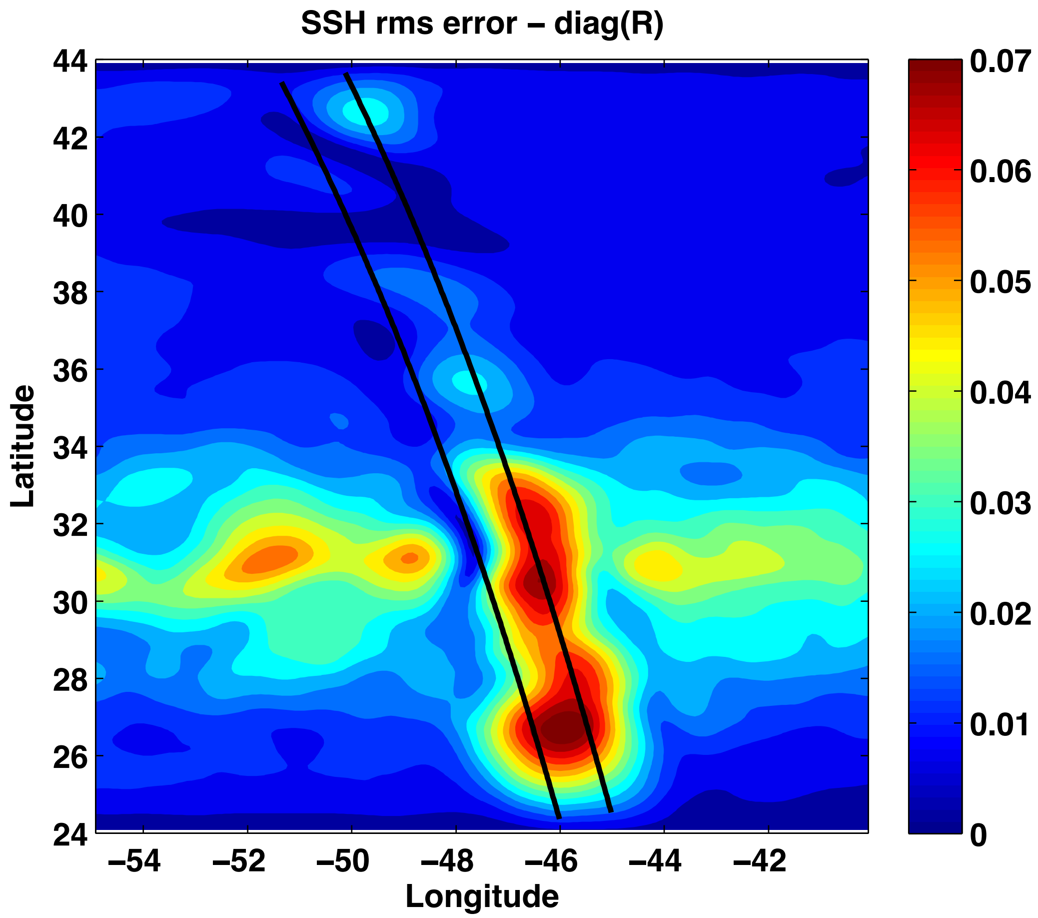
$$\left[\mathbf{y}^+ - \mathcal{H}^+(\mathbf{x})\right]^T \mathbf{R}^{+^{-1}} \left[\mathbf{y}^+ - \mathcal{H}^+(\mathbf{x})\right] = \left[\mathbf{y} - \mathcal{H}(\mathbf{x})\right]^T \mathbf{R}^{-1} \left[\mathbf{y} - \mathcal{H}(\mathbf{x})\right]$$

$$\mathbf{R^{-1}} = \mathbf{T^T R^{+^{-1}} T}$$

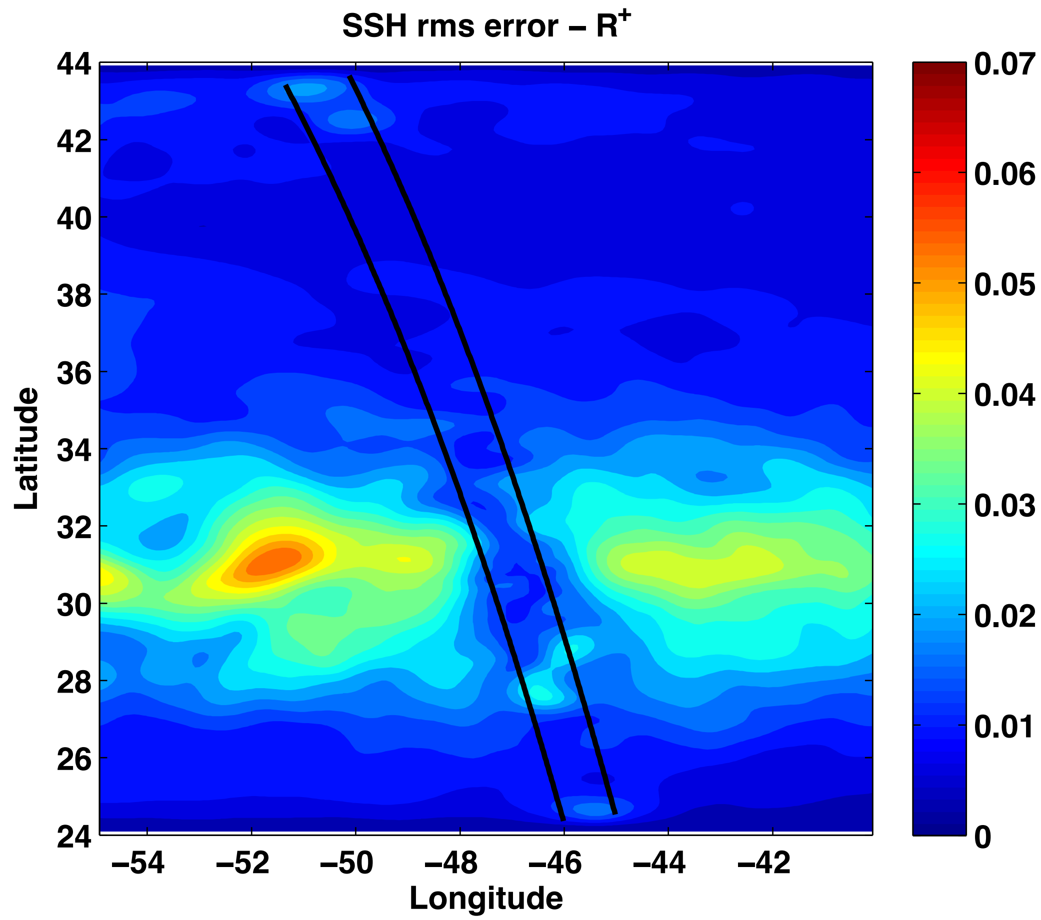
Result (Ruggiero et al, JAOT 2016)

- NEMO double gyre 1/12°
- RMS error after assimilation of one SWOT pass

Ignoring error correlations



Using R+ parameterization

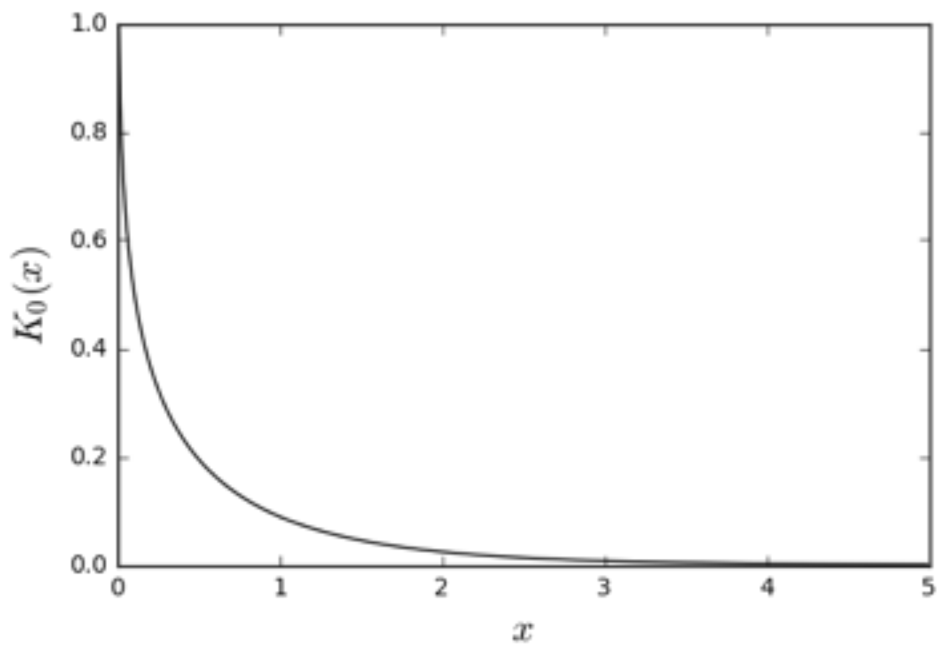


Limitation

This "R+" method can represent a limited number of correlation structures.

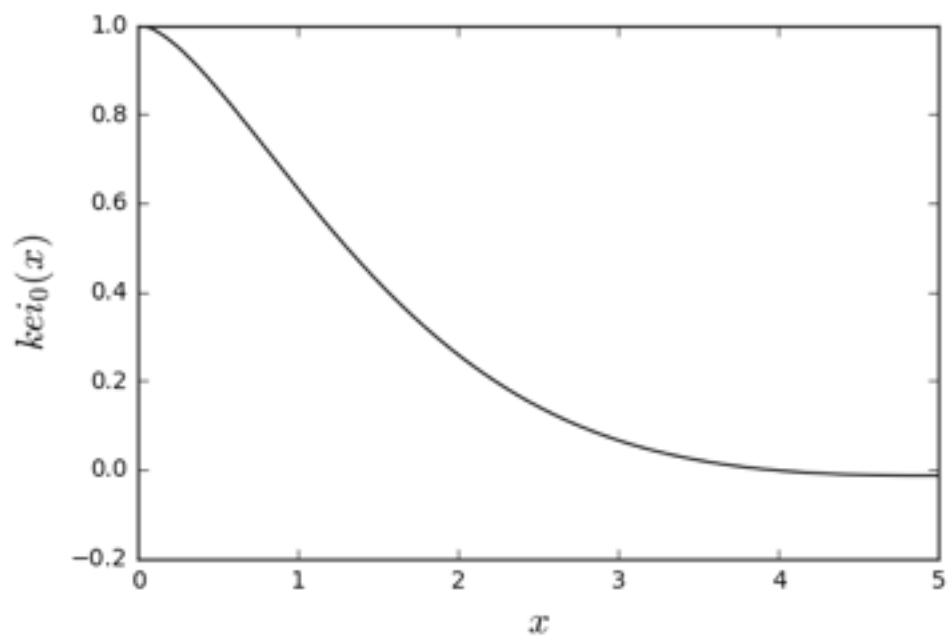
First-order derivatives

Bessel function of the second kind of order 0



Second-order derivatives

Kelvin function kei_0



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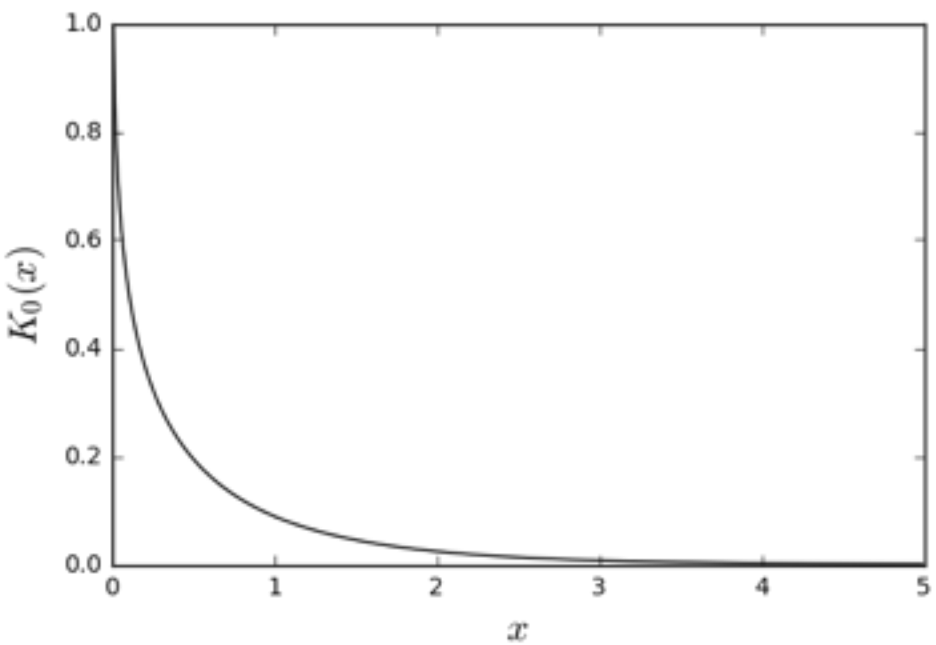
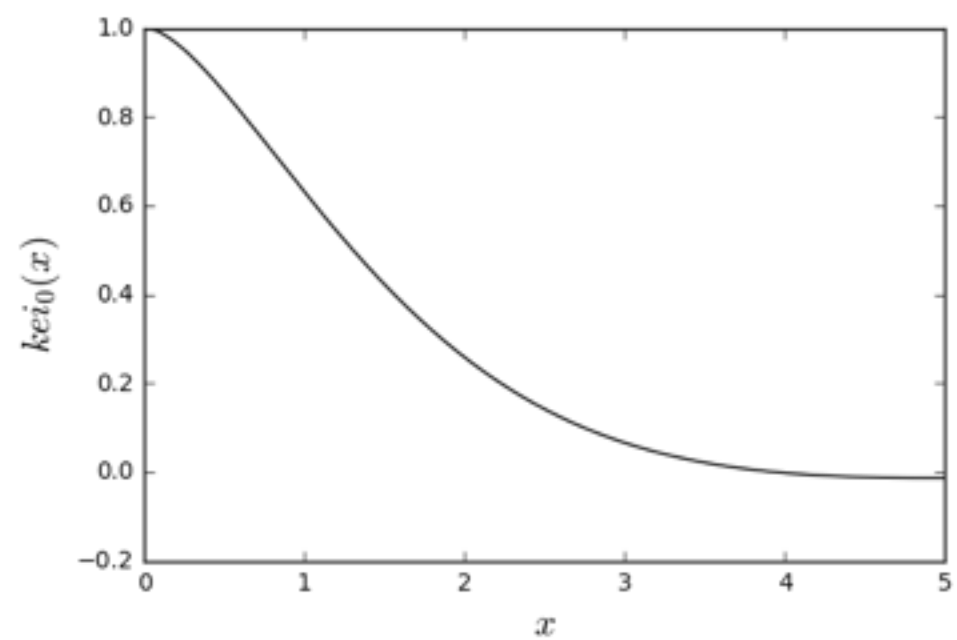


Second-order derivatives

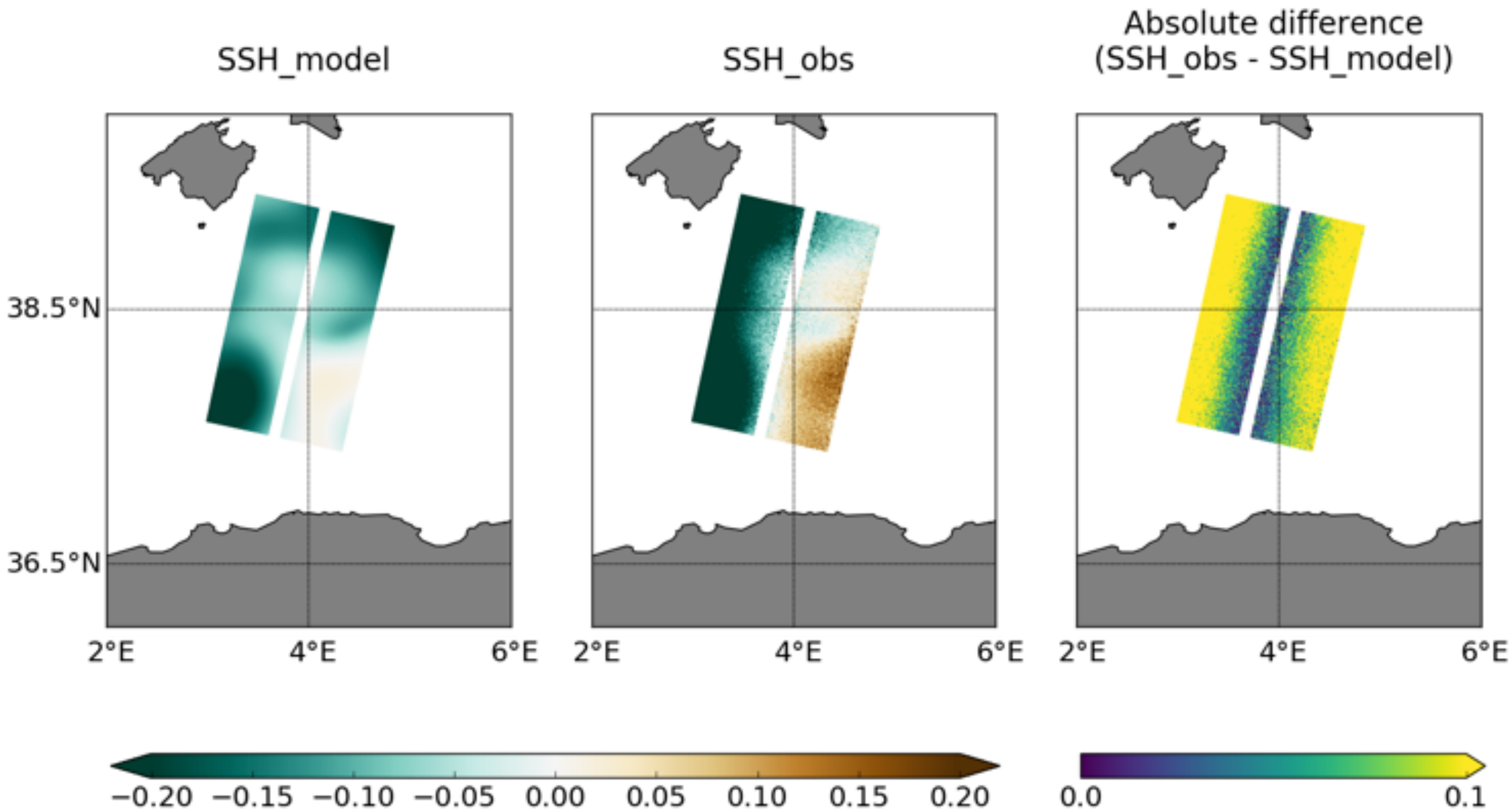
Kelvin function kei_0



We had to rule out uncorrelated (KaRIn) noise. The previous result was obtained with SWOT data averaged over 10x10 km² regions.

Denoising high resolution SWOT data for their assimilation

- Need to compute derivatives
- Not essential to remove ALL errors (correlated errors can be dealt with)

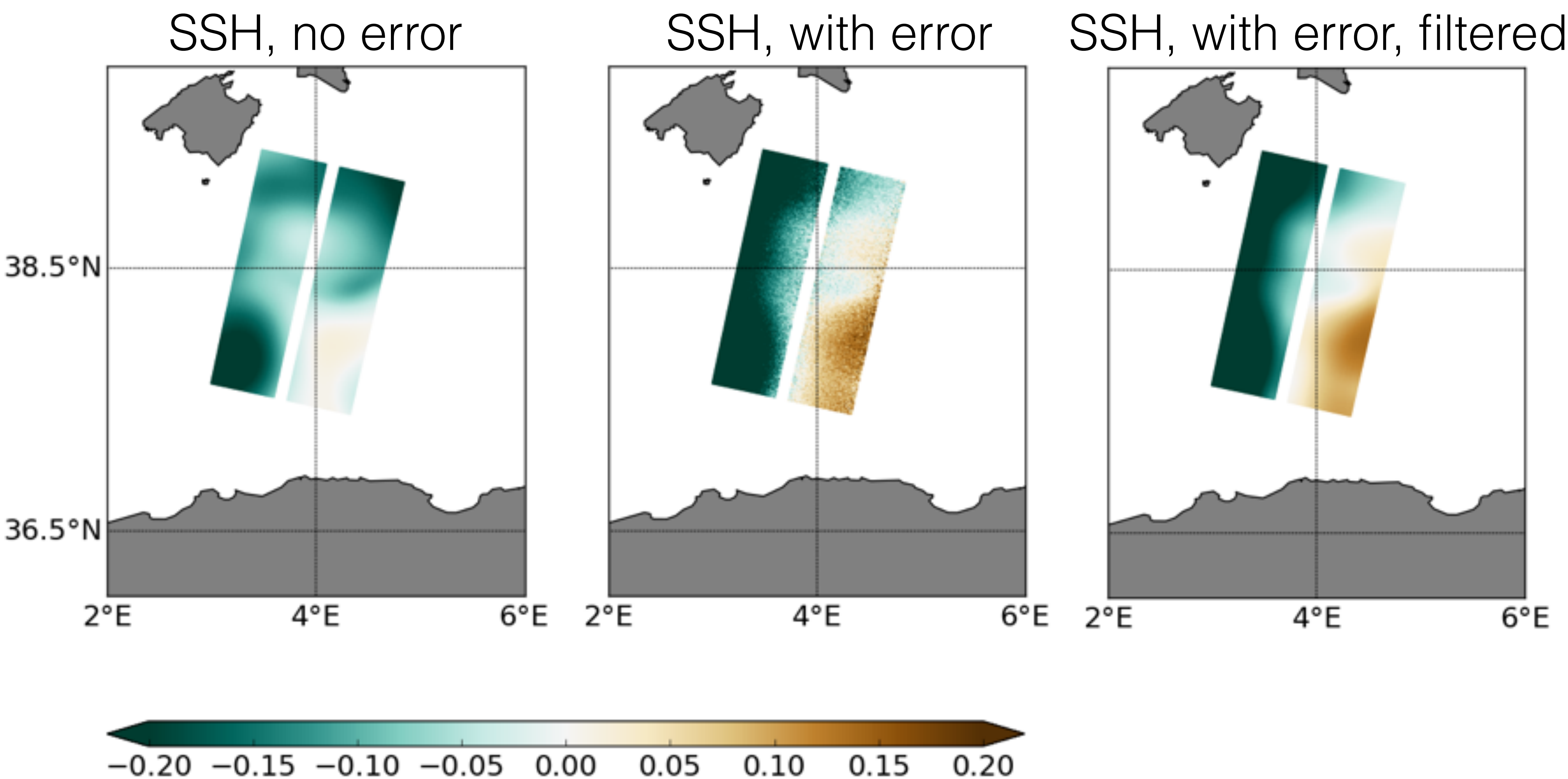


Denoising technique

J(h)=\frac12 ||h-h_{raw}||^2+\frac \lambda2||\nabla h||^2

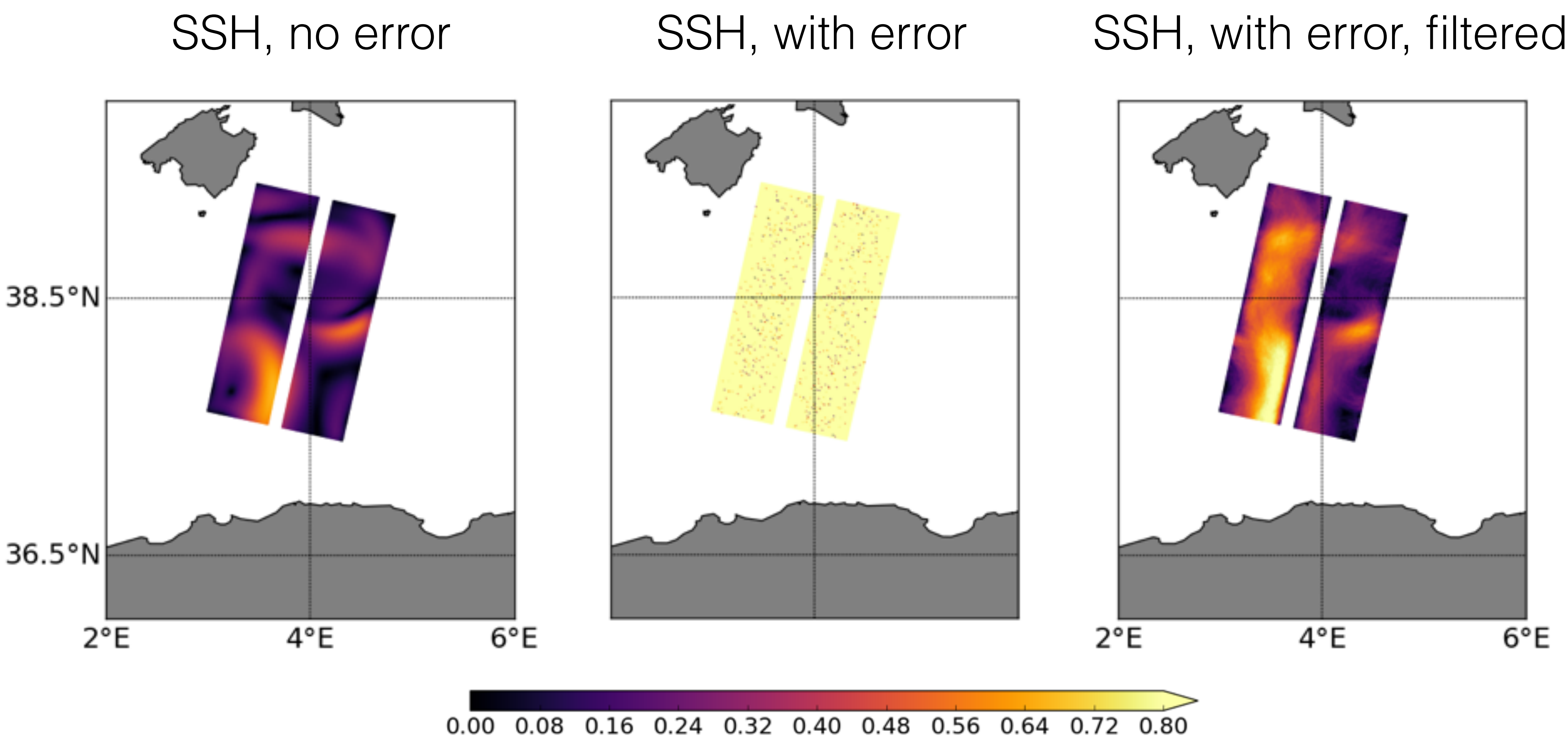
Penalize the gradients by minimizing:

$$J(h) = \frac{1}{2}||h - h_{raw}||^2 + \frac{\lambda}{2}||\nabla h||^2$$



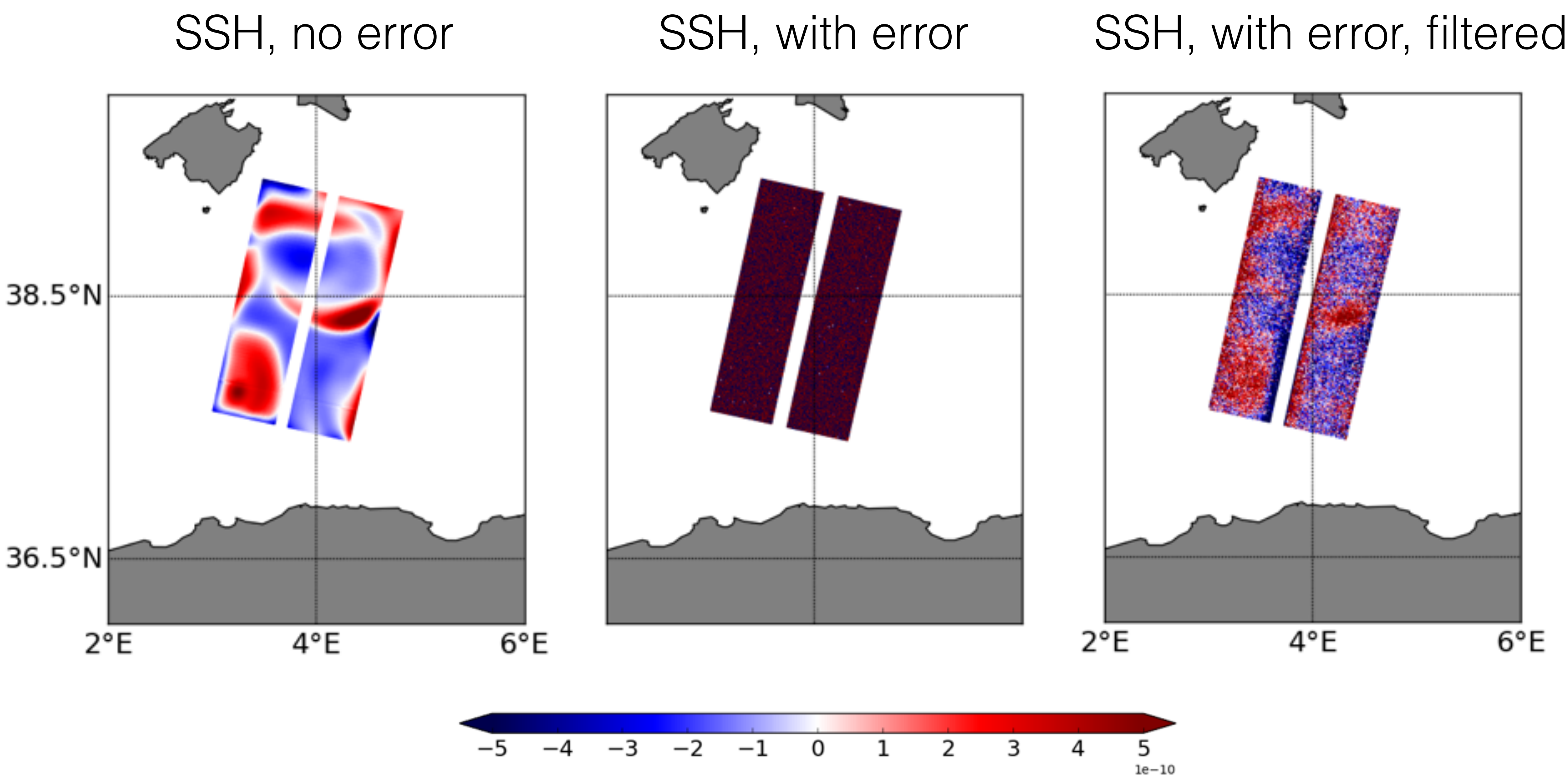
Denoising technique

Intensity of geostrophic velocity from:



Denoising technique

ΔSSH from:



Denoising technique

\frac\lambda2||\nabla \Delta h||^2

Next step: explore Tikhonov regularization term based on higher-order derivatives:

$$J(h) = \frac{1}{2}||h - h_{raw}||^2 + \cancel{\frac{\lambda}{2}||\nabla h||^2}$$

$$\frac{\lambda}{2}||\nabla \Delta h||^2 \qquad ?$$

Then tune the process (lambda, minimization, etc).

Extension: inpainting

J(h)=\frac12 ||m\cdot (h-h_{raw})||^2+
\frac\lambda2||\nabla h||^2

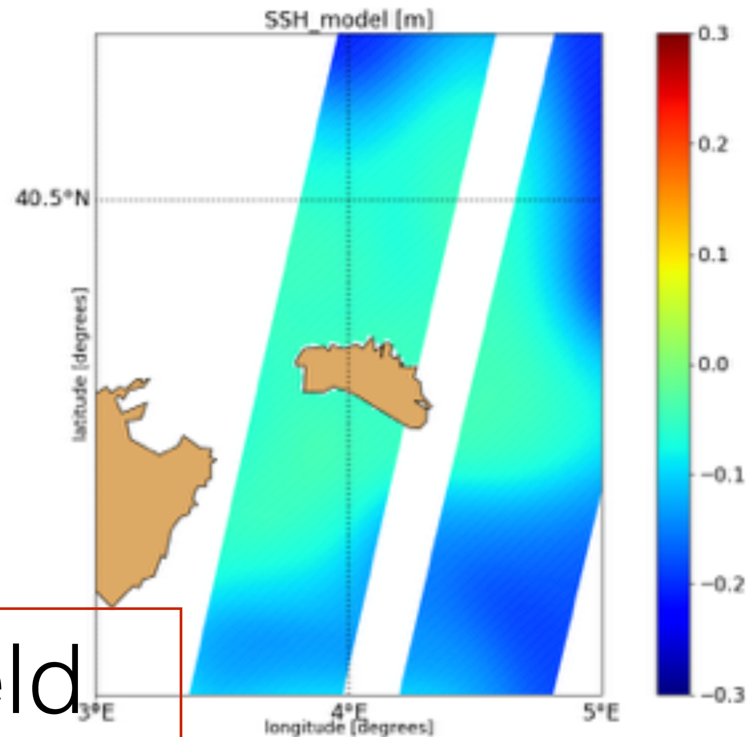
$$J(h) = \frac{1}{2}||\boxed{m\,\cdot}(h - h_{raw})||^2 + \frac{\lambda}{2}||\nabla h||^2$$

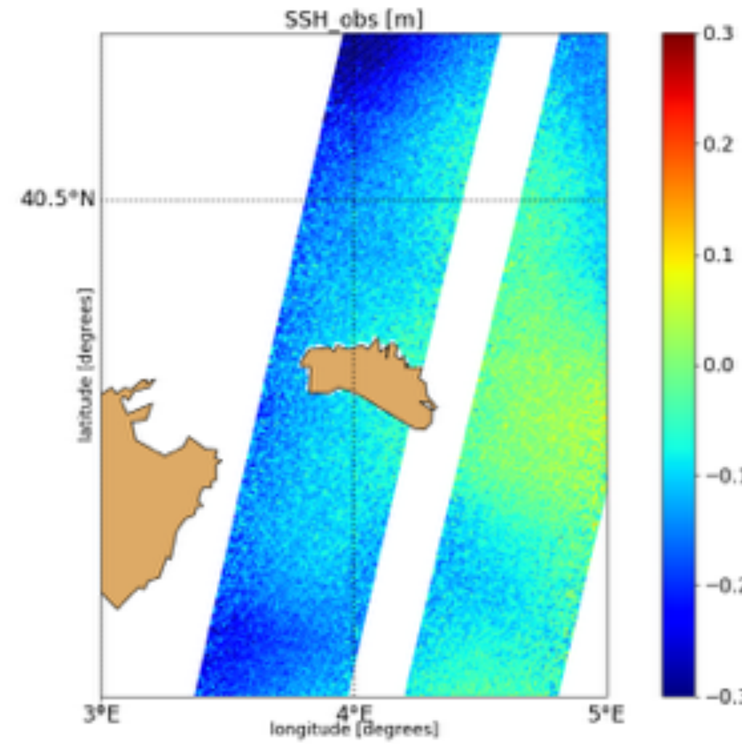
a mask to ignore pixels without data

Useful
- to deal with islands
- for data assimilation (?)

Extension: inpainting

Without noise



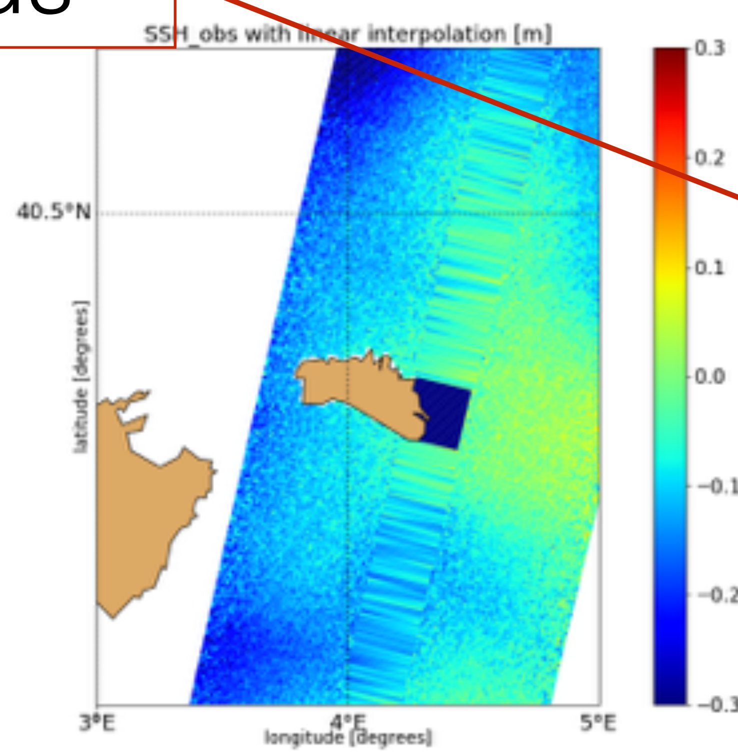


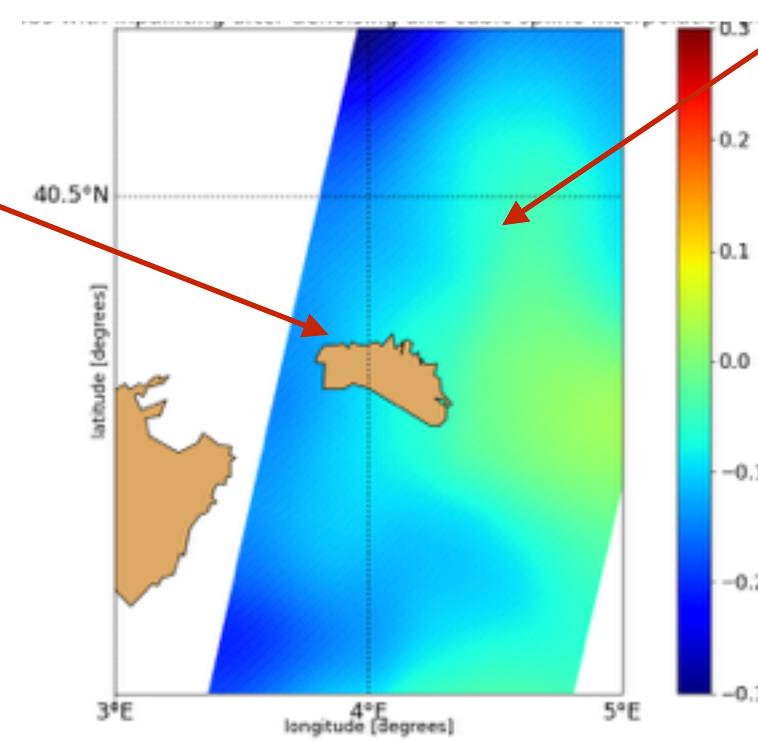
With noise

Smooth field near islands

Fill the gap

With noise, linear interpolation in the gap





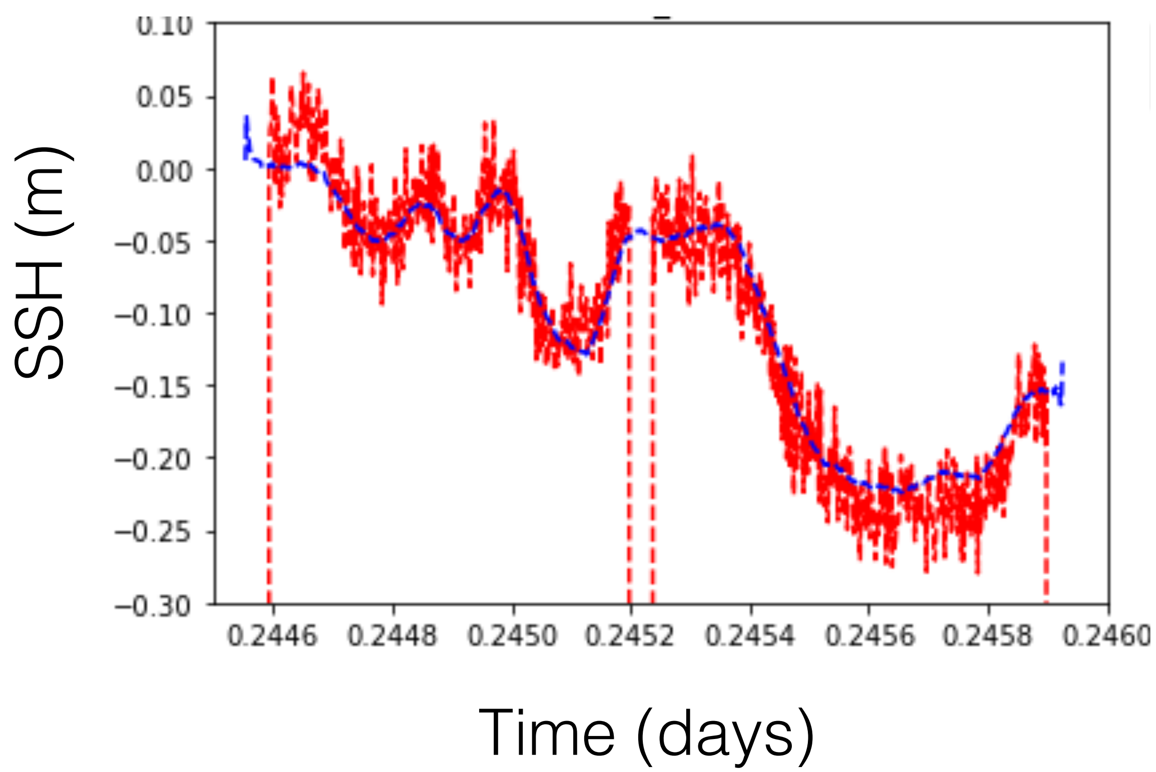
With noise, after denoising and inpainting

Extension: inpainting

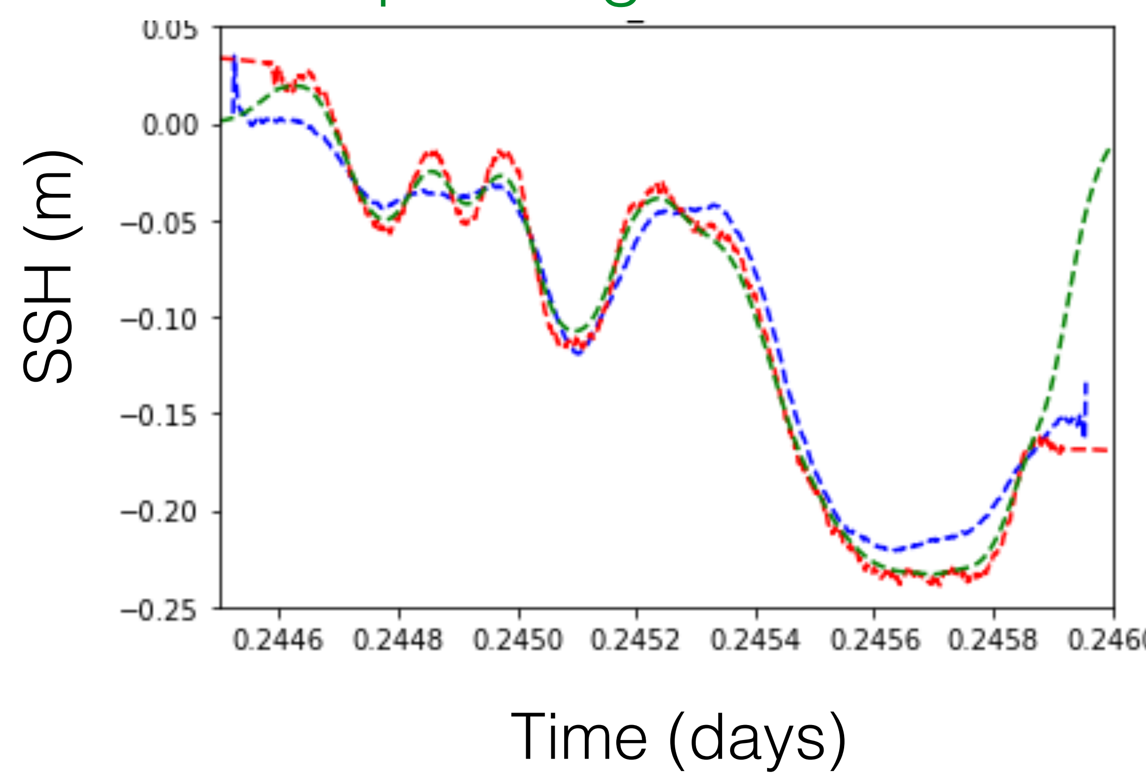
Nadir SSH along track

Blue: nadir SSH from simulator (smoothed)

Red: linear interp. from swath



Green: denoising
+inpainting from swath



Summary

We develop a method to remove small-scale noise from SWOT data, in the perspective of their correct assimilation.

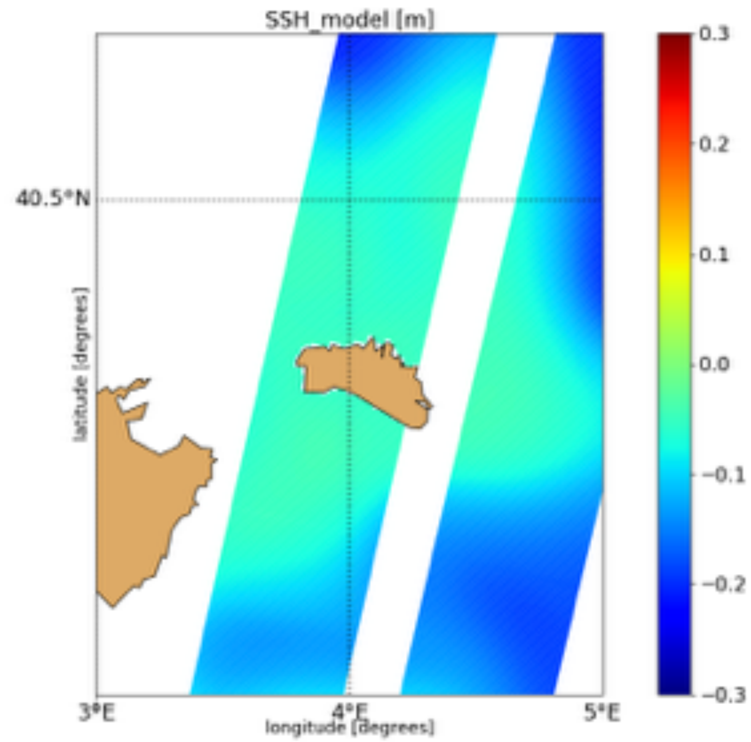
The method is based on a reduction of gradients and potentially higher order derivatives, consistently with the need of assimilation.

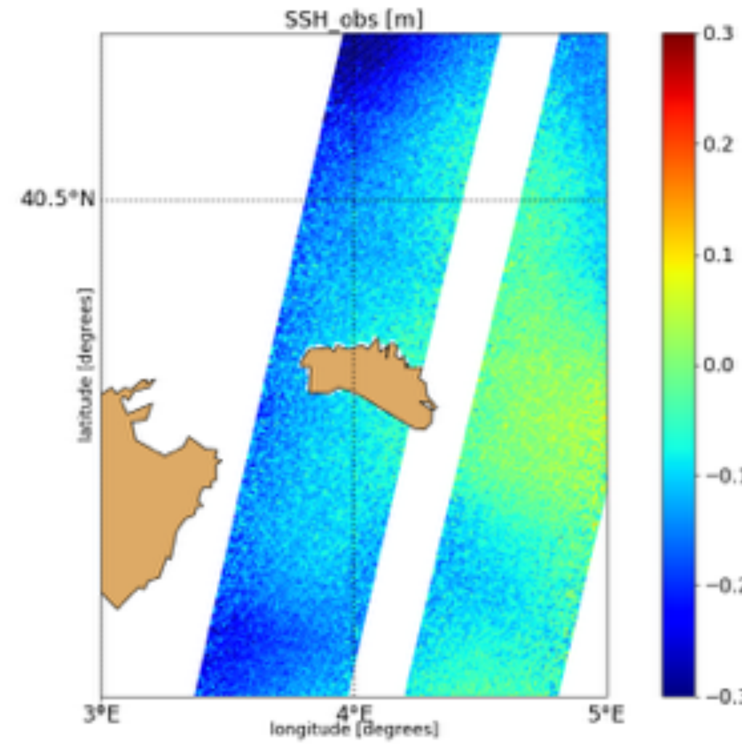
Inpainting comes with denoising. Useful to deal with islands and, perhaps, makes data assimilation simpler. Possible comparison with nadir data.

Work in progress.

Extension: inpainting

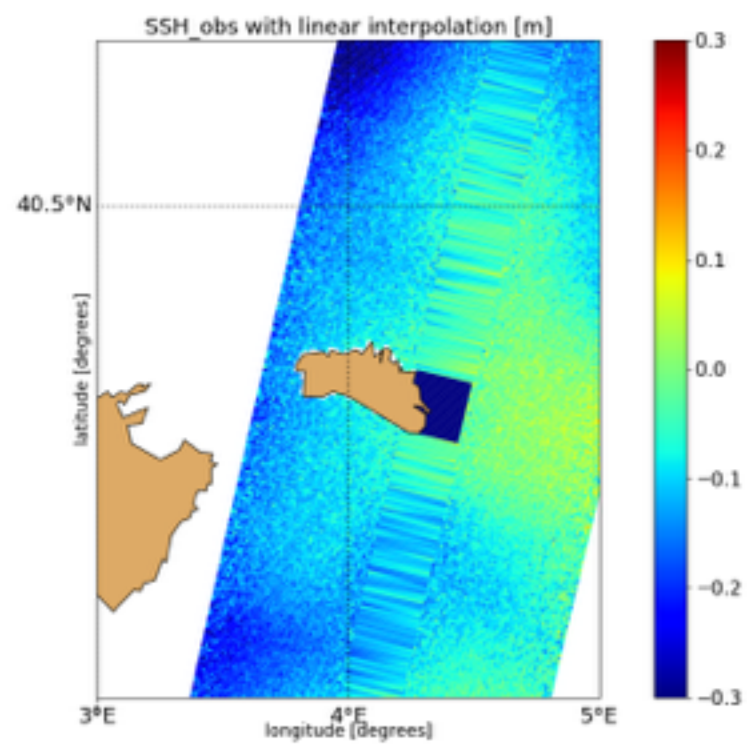
Without noise

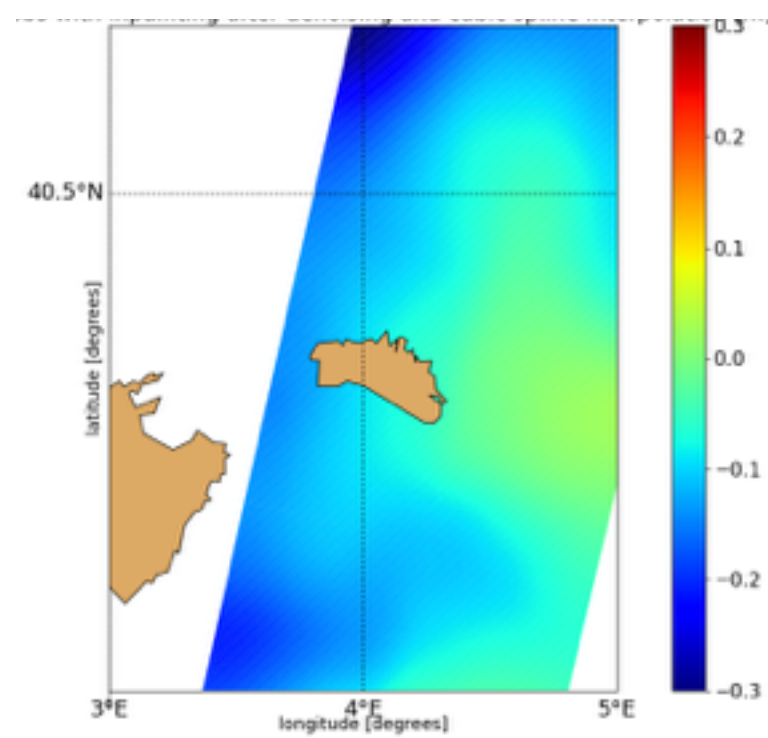




With noise

With noise, linear interpolation in the gap





With noise, after denoising and inpainting in the gap

Introduction