On a dynamical separation of waves and balanced motions based on surface measurements

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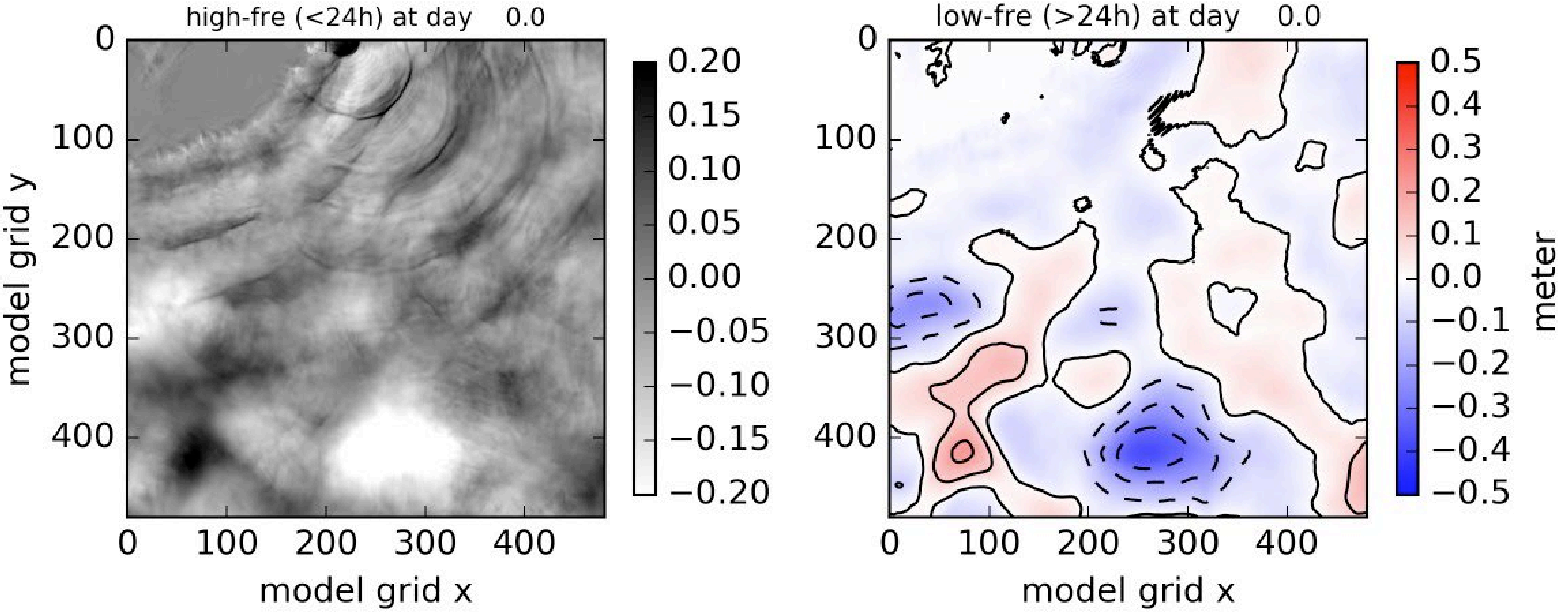
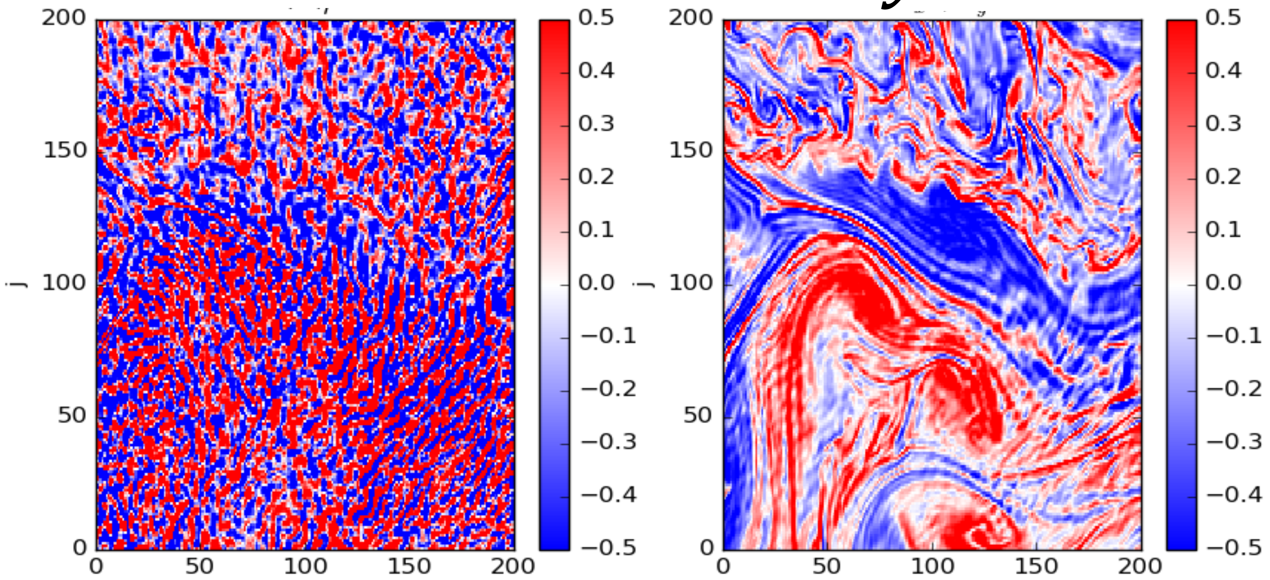
The coexistence of the waves and balanced motions in SSH



SSH from 1/48 MITgcm simulation

The simple Laplacian of SSH is not relative vorticity.

$g\,\nabla^2\eta/f^2$ $\qquad$ $-u_y + v_x$



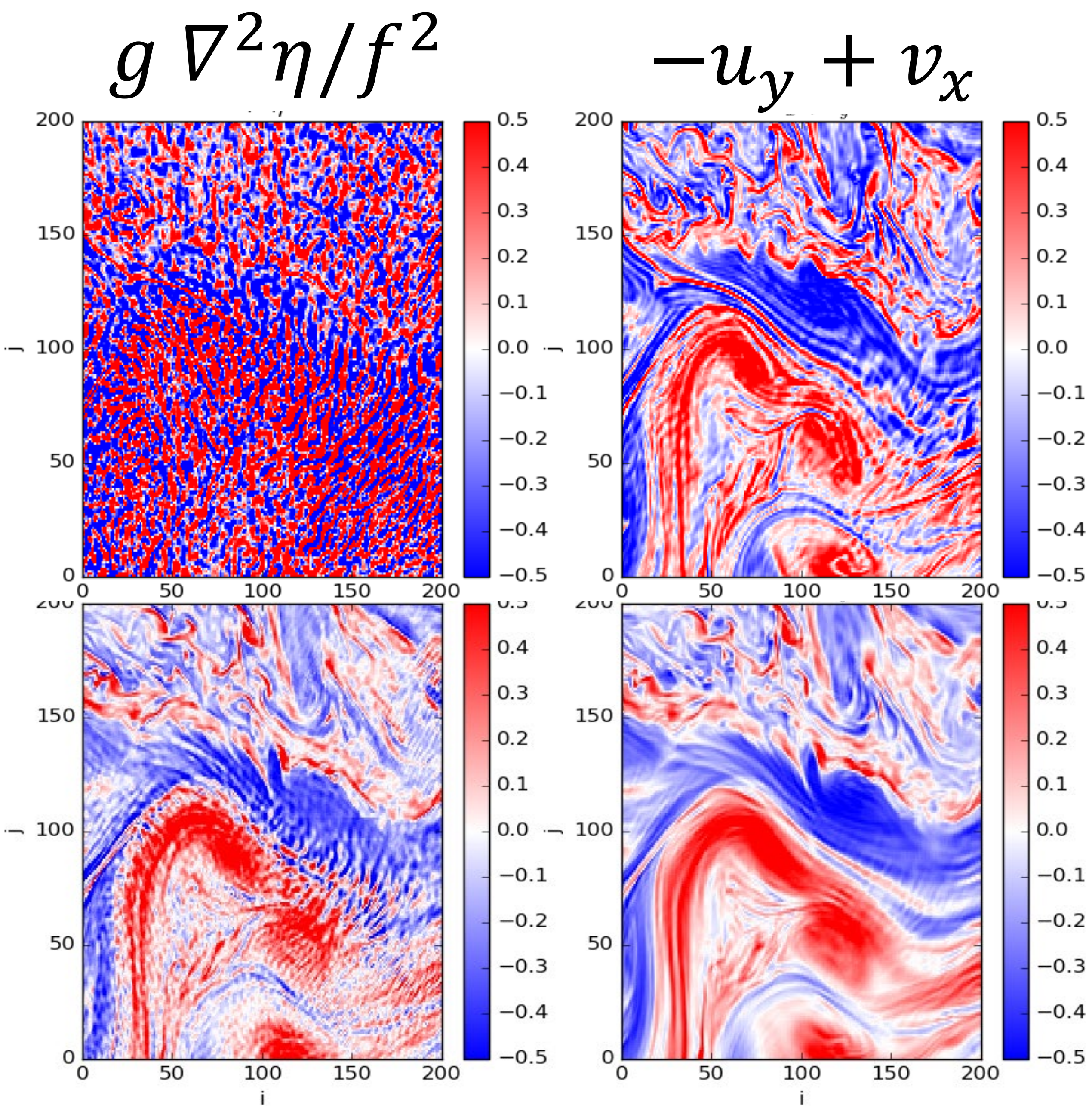
Snapshot

Q: How to separate internal waves and balanced motions?

A: Daily average.

The simple Laplacian of SSH is not relative vorticity.

Daily average does yield results approaching balanced dynamics.



Snapshot

Daily average

Can we regenerate the SSH time evolution based on satellite snapshots?

What are the requirements?

We need an initial condition with

$$\eta(x, y)$$
$$\rho(x, y, z)$$
$$\vec{u}_H(x, y, z)$$

interior+surface Quasi-geostrophy+Waves

Based on Wang et al. (2013) and Ponte et al. (2017)

Interior PV

$$\eta = \eta^{sg} + \eta^{ig} + \eta^{iw}$$

SQG

waves

SST

Share the QG dynamics

Share the baroclinic normal mode basis

Wang et al. (2013): $\eta^{ig} = \eta - \eta^{sg}$

Does not distinguish η^{ig} *and* η^{iw}

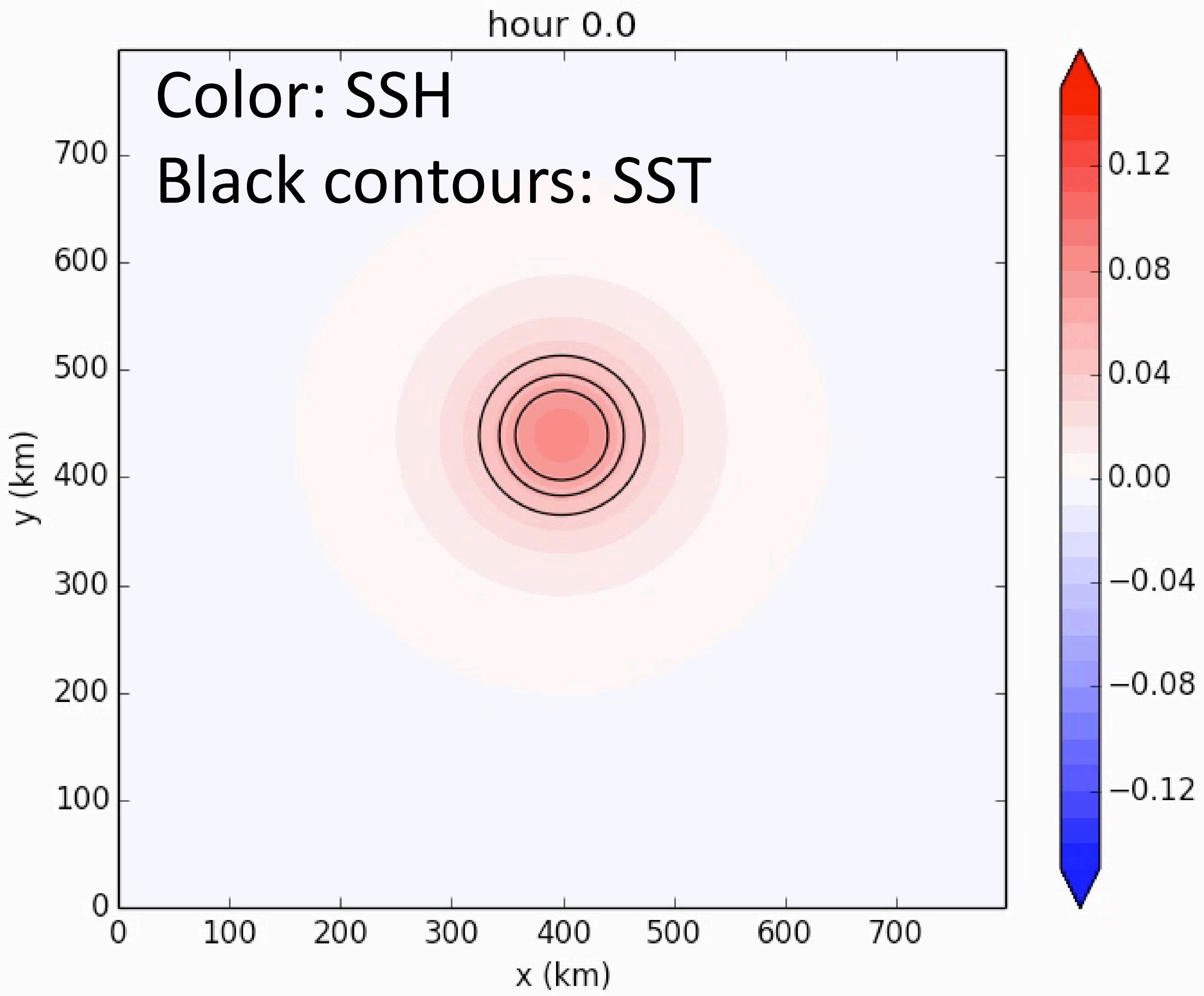
Ponte et al. (2017):

$\eta^{iw} = \eta - \eta^{sg}(eSQG)$

Miss the interior PV not resolved by eSQG

An idealized demo the control run

$$\omega = 1.41\times 10^{-4} s^{-1}$$
$$\mathrm{f} = 8\times 10^{-5} s^{-1}$$
$$H = -5000m$$
$$dx = dy = 2km$$
$$L_x = L_y = 800km$$
$$R_d = 70km$$
$$F^y = f^{-1} U_0 \sin(\omega t)\, F_1(z)$$
$$at\, y = 50km$$
$$\rho' = \rho'_0 \exp\left(-\frac{r^2}{60^2 km^2}\right)\phi(z)$$



Setup similar to Dunphy and Lamb (2014)

An idealized demonstration

Not considered for now

Interior QG

$$\eta = \eta^{sg} + \eta^{ig} + \eta^{iw}$$

SQG waves

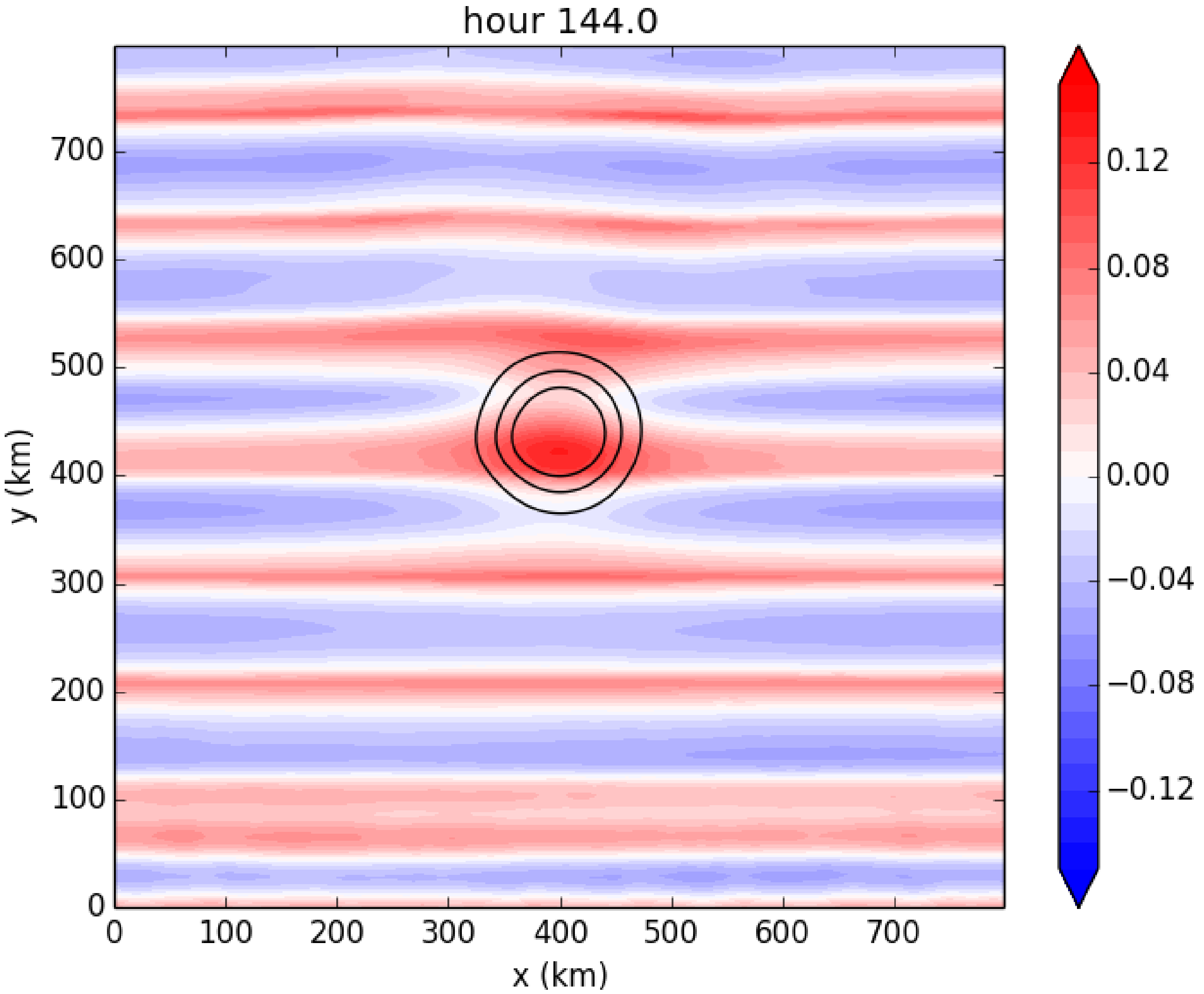
Initial condition:

SSH: η

Density: $\rho = \rho^{sg} + \rho^{iw}$

Horizontal velocity:

$$\boldsymbol{u_H} = \boldsymbol{u_H^{sg}} + \boldsymbol{u_H^{iw}}$$



$\boldsymbol{\eta}$ to $\vec{\boldsymbol{u}}_{\boldsymbol{H}}$ is the tricky part

The basis of using SSH as a dynamical variable:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta'}{\partial x}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta'}{\partial y}$$

$$\eta' = \eta'\left[i(kx + ly - \omega t + \theta_0)\right]$$

$$u = \frac{k\omega + ilf}{\omega^2 - f^2} g\eta'$$

$$v = \frac{l\omega - ikf}{\omega^2 - f^2} g\eta'$$

$$\eta = \eta^{sg} + \eta^{iw}$$

SQG waves

$$\omega \ll f: geostrophic\ approximation: \ u = -\frac{g\eta_y}{f}, v = \frac{g\eta_x}{f}$$

$$\omega \geq f: inertia - gravity\ waves$$

Gill 1982, pp.257-263

$\boldsymbol{\eta}$ to $\vec{\boldsymbol{u}}_{\boldsymbol{H}}$ is the tricky part

The basis of using SSH as a dynamical variable:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta^{iw}}{\partial x}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta^{iw}}{\partial y}$$

unknown

$$\eta^{iw} = \hat{\eta}^{iw}\left[i(kx + ly - \omega t + \theta_0)\right]$$

$$\hat{u} = \frac{k\omega + ilf}{\omega^2 - f^2} g\hat{\eta}^{iw}$$

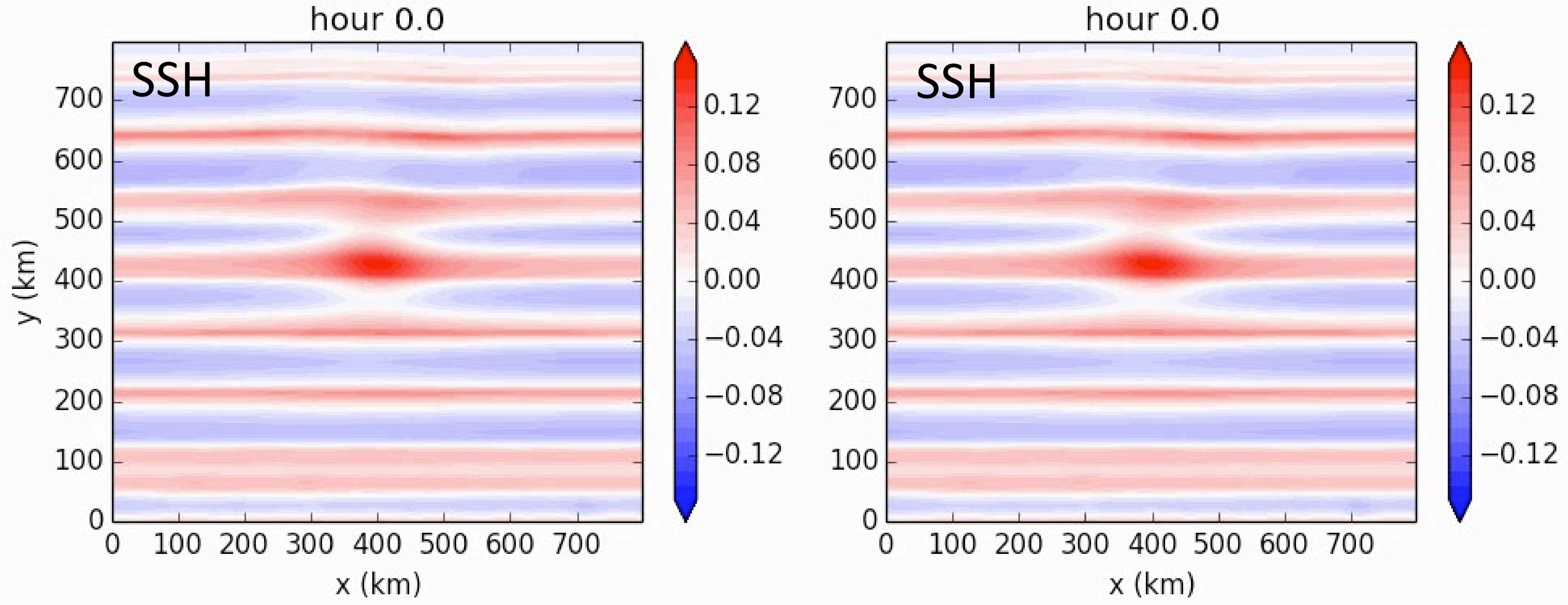
$$\hat{v} = \frac{l\omega - ikf}{\omega^2 - f^2} g\hat{\eta}^{iw}$$

$$\omega \ll f: geostrophic\ approximation: \ u^{sg} = -\frac{g\eta_y^{sg}}{f}, v^{sg} = \frac{g\eta_x^{sg}}{f}$$

$$\omega \geq f: inertia - gravity\ waves$$

Gill 1982, pp.257-263

$$\eta;\ \rho = \rho^{sg} + \rho^{iw};\ \boldsymbol{u_H} = \boldsymbol{u_H^{sg}} + \boldsymbol{u_H^{iw}}$$



Cumulative time average since hour 0

$\boldsymbol{\eta}$ to $\vec{\boldsymbol{u}}_{\boldsymbol{H}}$ is the tricky part

The basis of using SSH as a dynamical variable:

$$\frac{\partial u}{\partial \cdots} \quad f \cdots \quad \partial \eta^{iw} \qquad k\omega + ilf$$

inaccuracy in velocity reconstruction

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y}$$

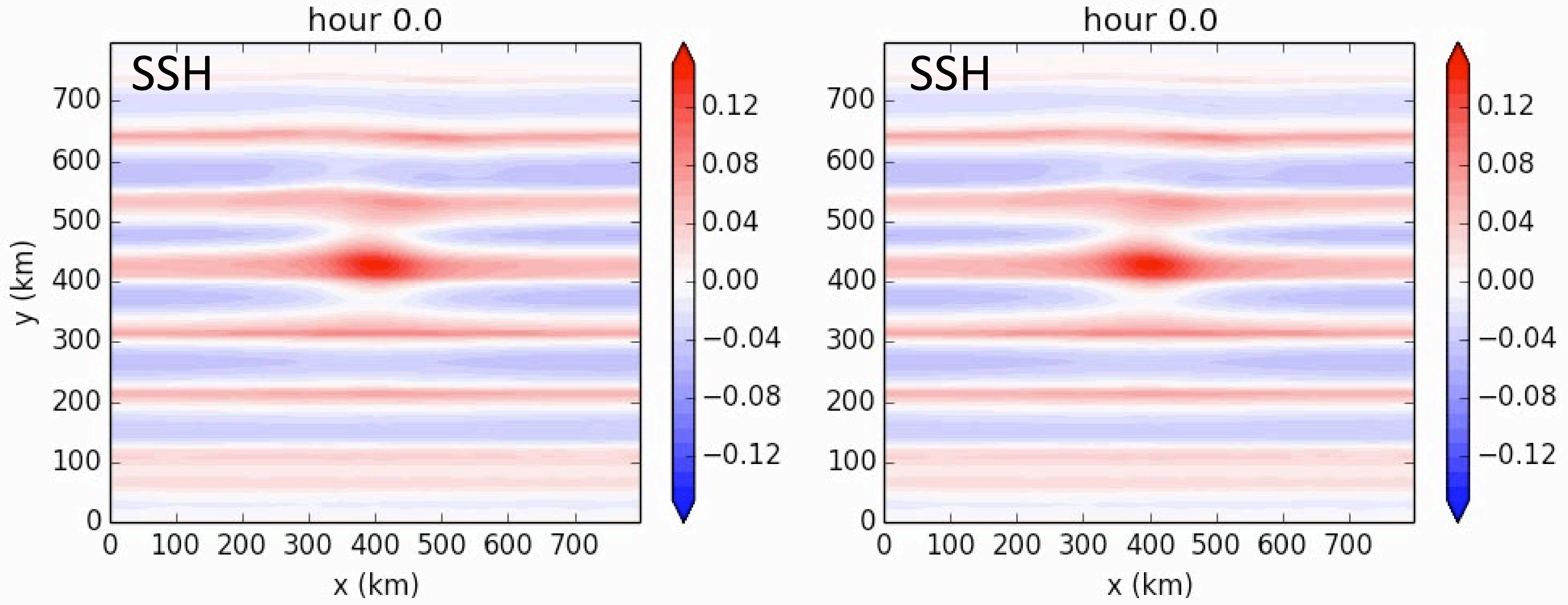
$$\eta^{iw} = \hat{\eta}^{iw}\,[i(kx + ly - \omega t + \theta_0)] \qquad \hat{v} = \frac{l\omega - ikf}{\omega^2 - f^2}\, g\hat{\eta}^{iw}$$

$$\omega \ll f: geostrophic\ approximation:\ \ u^{sg} = -\frac{g\eta_y^{sg}}{f}, v^{sg} = \frac{g\eta_x^{sg}}{f}$$

$$\omega \geq f: inertia - gravity\ waves$$

Gill 1982, pp.257-263

$$\eta;\ \rho = \rho^{sg} + \rho^{iw};\ \boldsymbol{u_H} = \boldsymbol{u_H^{sg}} + \boldsymbol{u_H^{iw}}\ \boldsymbol{u_H}(z=0)F_1(z)$$



Cumulative time average since hour 0

Conclusions and discussions

$$\eta = \eta^{sg} + \eta^{ig} + \eta^{iw}$$

- SSH, SST, sea surface velocity all together can produce accurate initial condition for regenerating the time evolution of SSH.

- High resolution SST measurements are not always accessible. More interior data such as Argo measurements will help better reconstruct the vertical structure.
- It is deterministic in this study but an optimization problem in reality. The framework can be used to design a simplified dynamic core for high-resolution data assimilation.
- Is it possible to distinguish η^{ig} and η^{iw} from SSH snapshots? (we need new insight).

One more step towards the reality

work in progress

$$\eta = \eta^{sq} + \eta^{ig} + \eta^{iw}$$

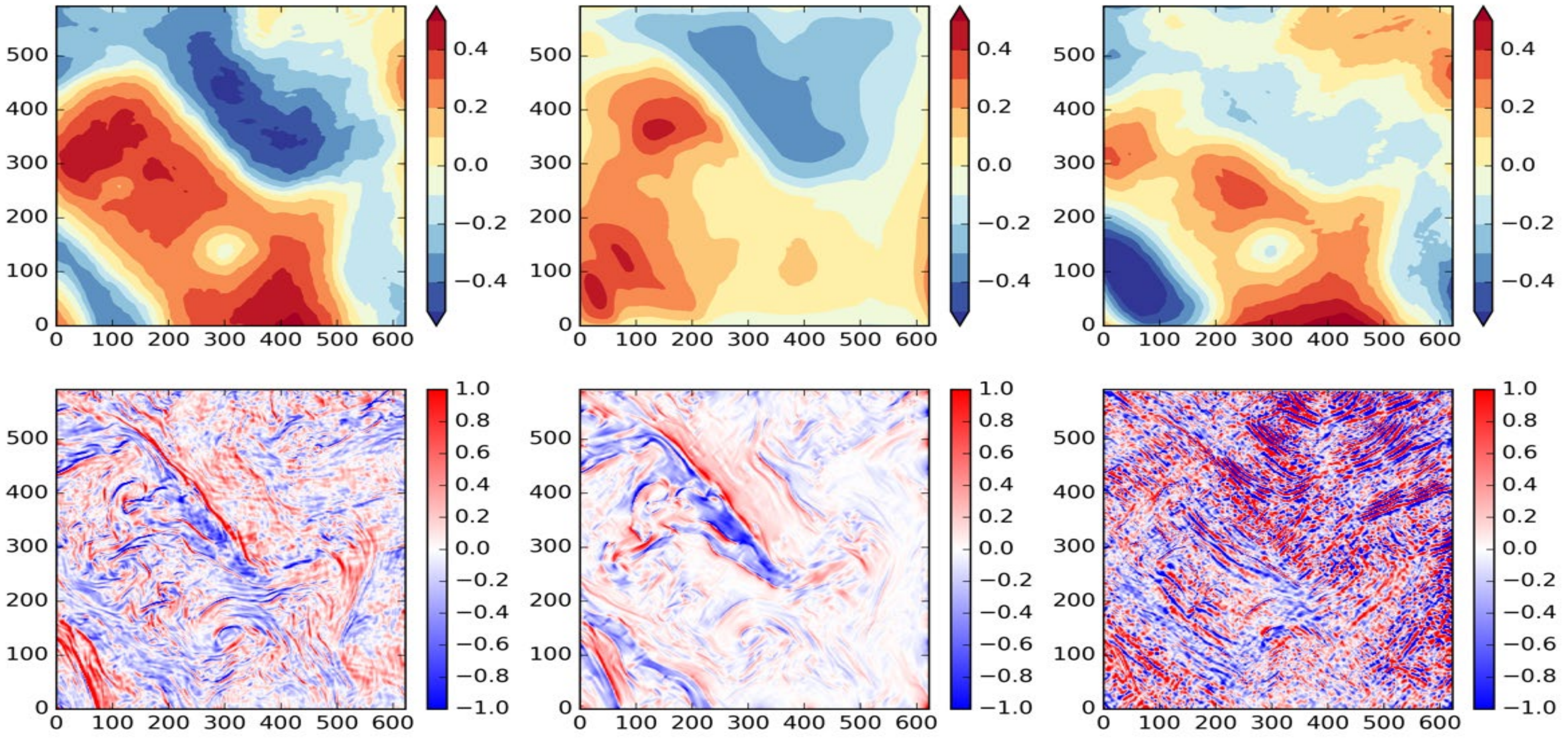
Based on $\frac{1}{48}^{\circ}$ MITgcm

Interior balanced mode is mixed in $\eta - \eta^{sq}$

$$\frac{\partial \zeta}{\partial t} = f \frac{\partial w}{\partial z}$$

Relative vorticity is influenced by internal waves through stretching

η η^{sg} $\eta^{ig} + \eta^{iw}$



ζ/f

True relative vorticity based on velocity

$\frac{g}{f^2}\nabla^2\eta^{sg}$

$\frac{g}{f^2}\nabla^2(\eta^{ig}+\eta^{iw})$

Not vorticity

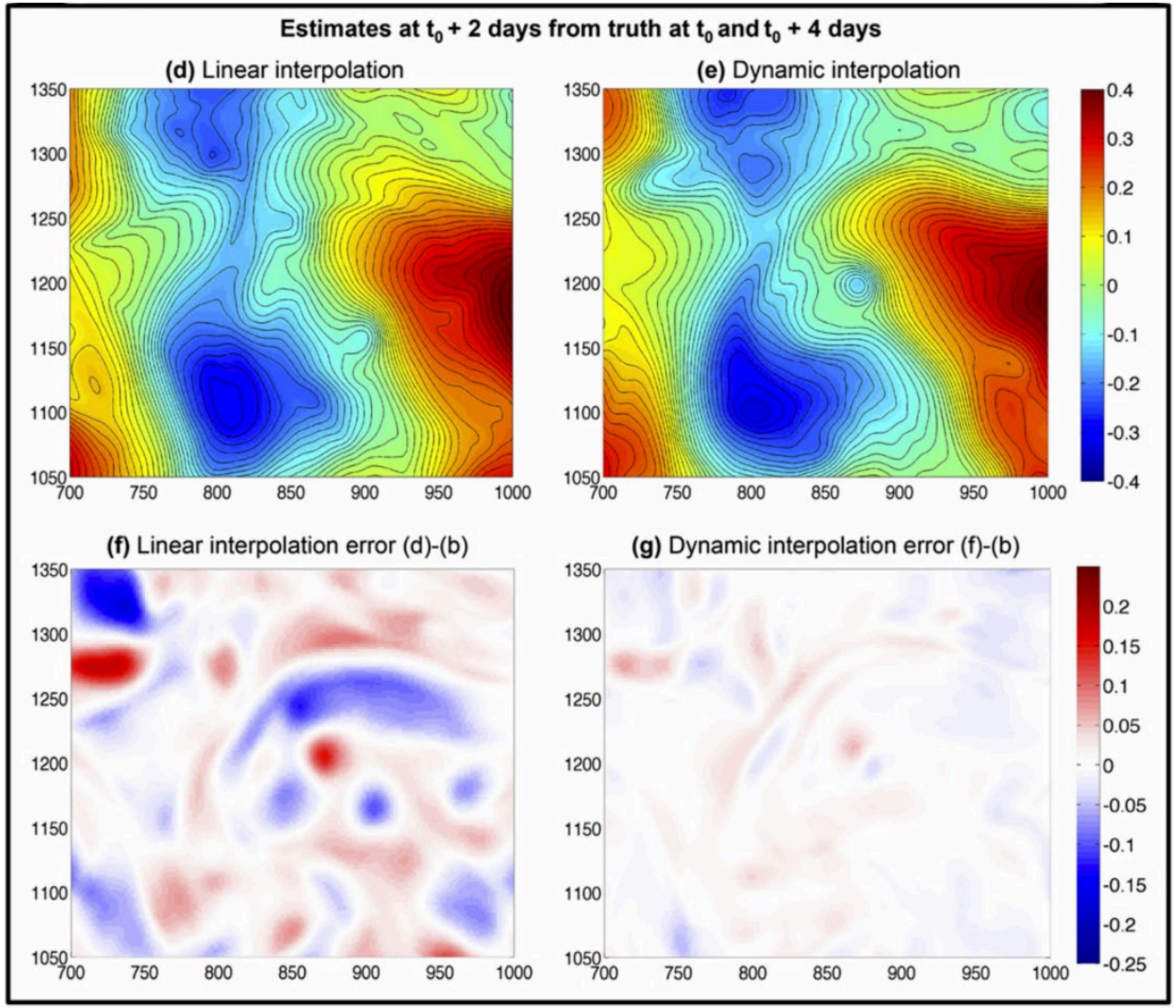
Challenge 1. Gap filling

Dynamic interpolation
(Ubelmann et al. 2015,2016)

Use the 1½ layer QG model to horizontally advect information to fill the temporal gaps (PV dynamics)

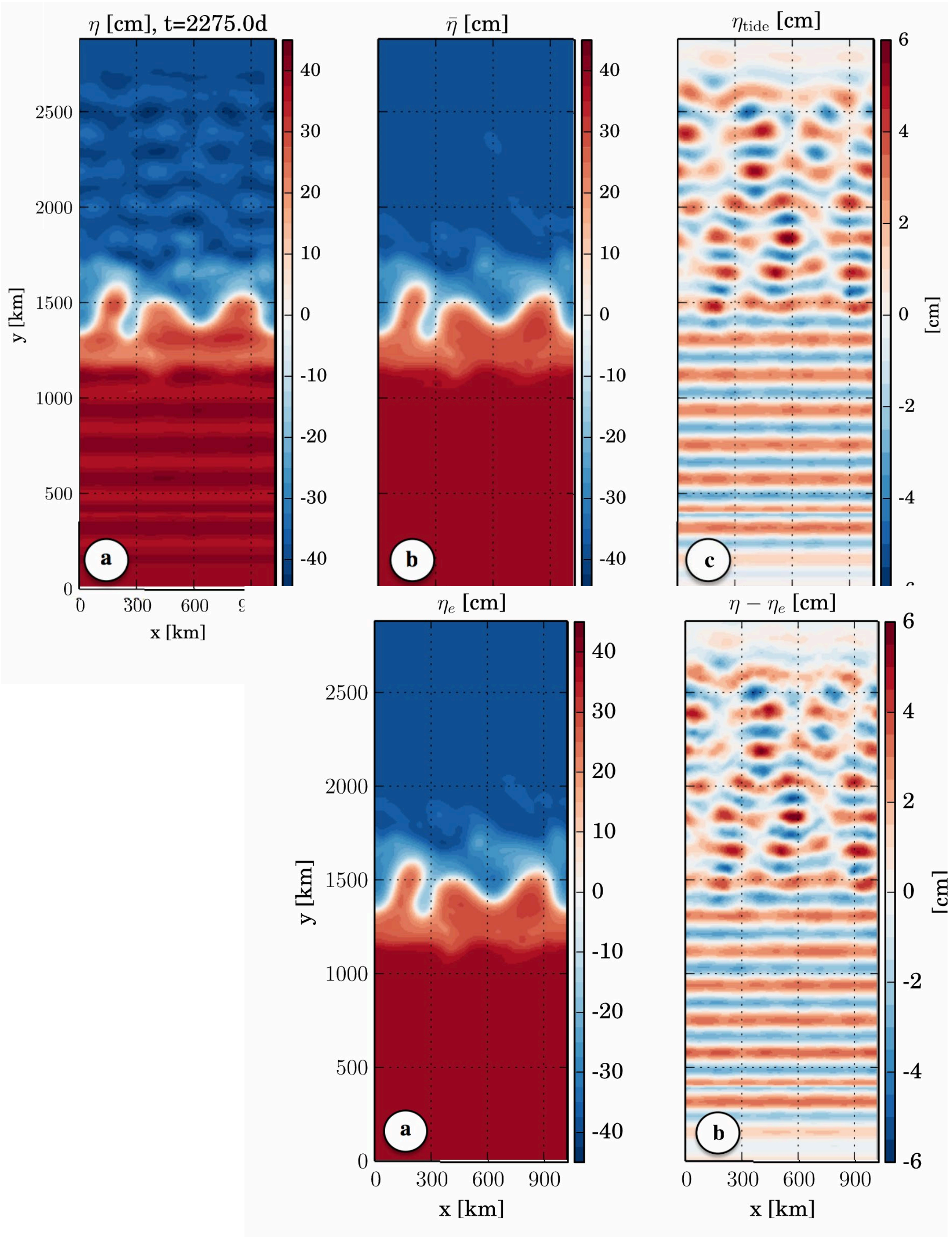
Remaining challenges:

1. Account for the surface PV contribution (SQG)
2. Capture the multi-modal physics
3. Include unbalanced high frequency internal waves



Wave-eddy separation from SSH snapshots Ponte et al. (2017)

1. Surface density for balanced motion (eSQG framework)
2. The residual is due to waves

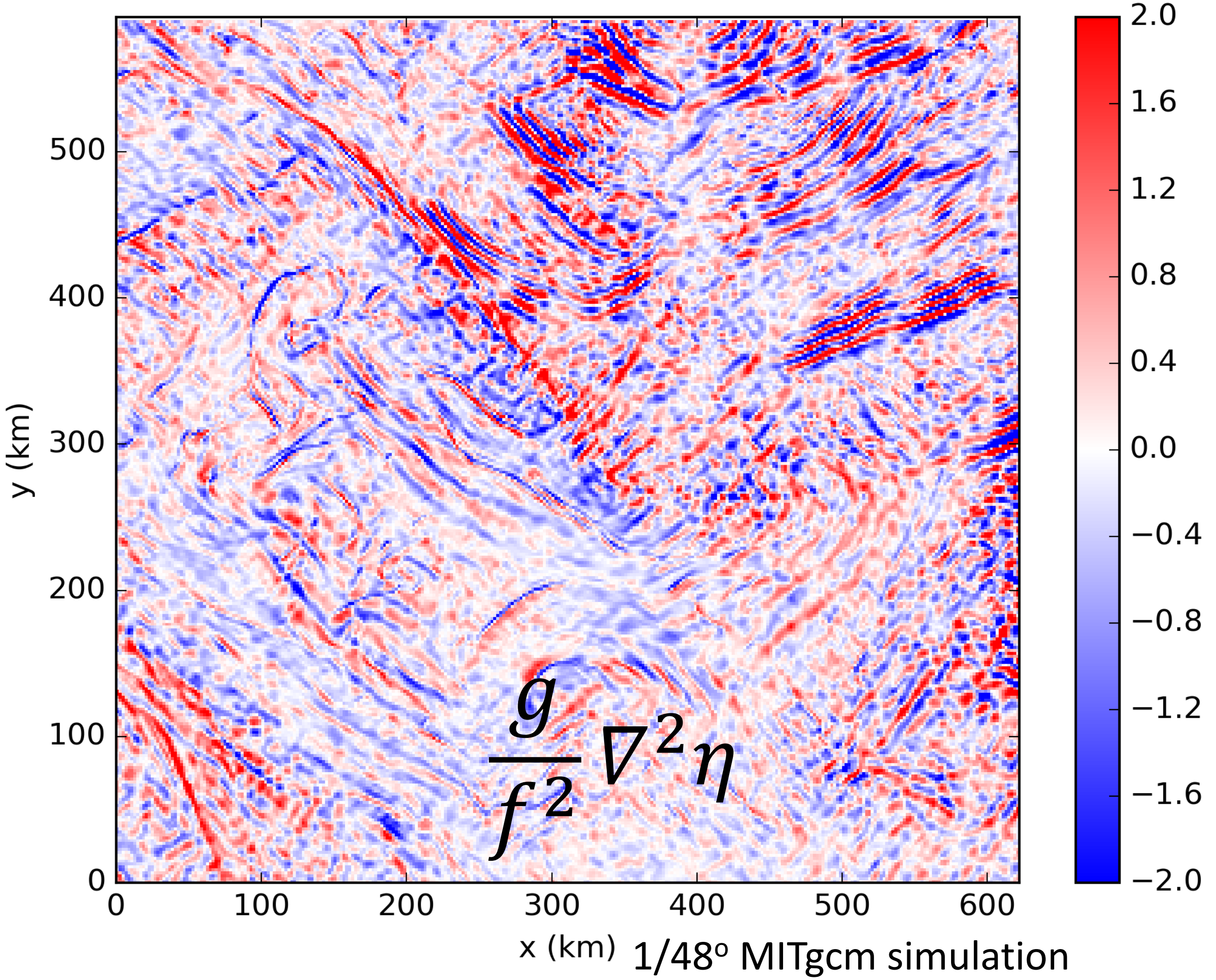


The breakdown of geostrophic balance in SSH due to high frequency waves

$$\nabla^2\psi \neq \zeta$$

The simple Laplacian of SSH is not relative vorticity.

pressure potential contributes to the velocity tendency



1/48° MITgcm simulation

Reconstruction in the two dynamical regimes

$\omega \ll f, rotational\ component$

$\omega \geq f, propogating\ component$

QG dynamics

$$\frac{dq}{dt} = 0$$

$$q = \nabla^2\psi + \frac{f^2}{N^2}\partial_{zz}\psi$$

$$u = -\psi_y, v = \psi_x$$

$$\partial_z\psi = \frac{\rho'}{f}$$

$$\nabla^2\psi + \frac{f^2}{N^2}\partial_{zz}\psi = Q$$

Wave dynamics

$$(\partial_t^2 + f^2)\partial_z^2 w + \left[N^2(z) + \partial_t^2\right]\left(\partial_x^2 + \partial_y^2\right)w = 0$$

Polarization relations with a constant N

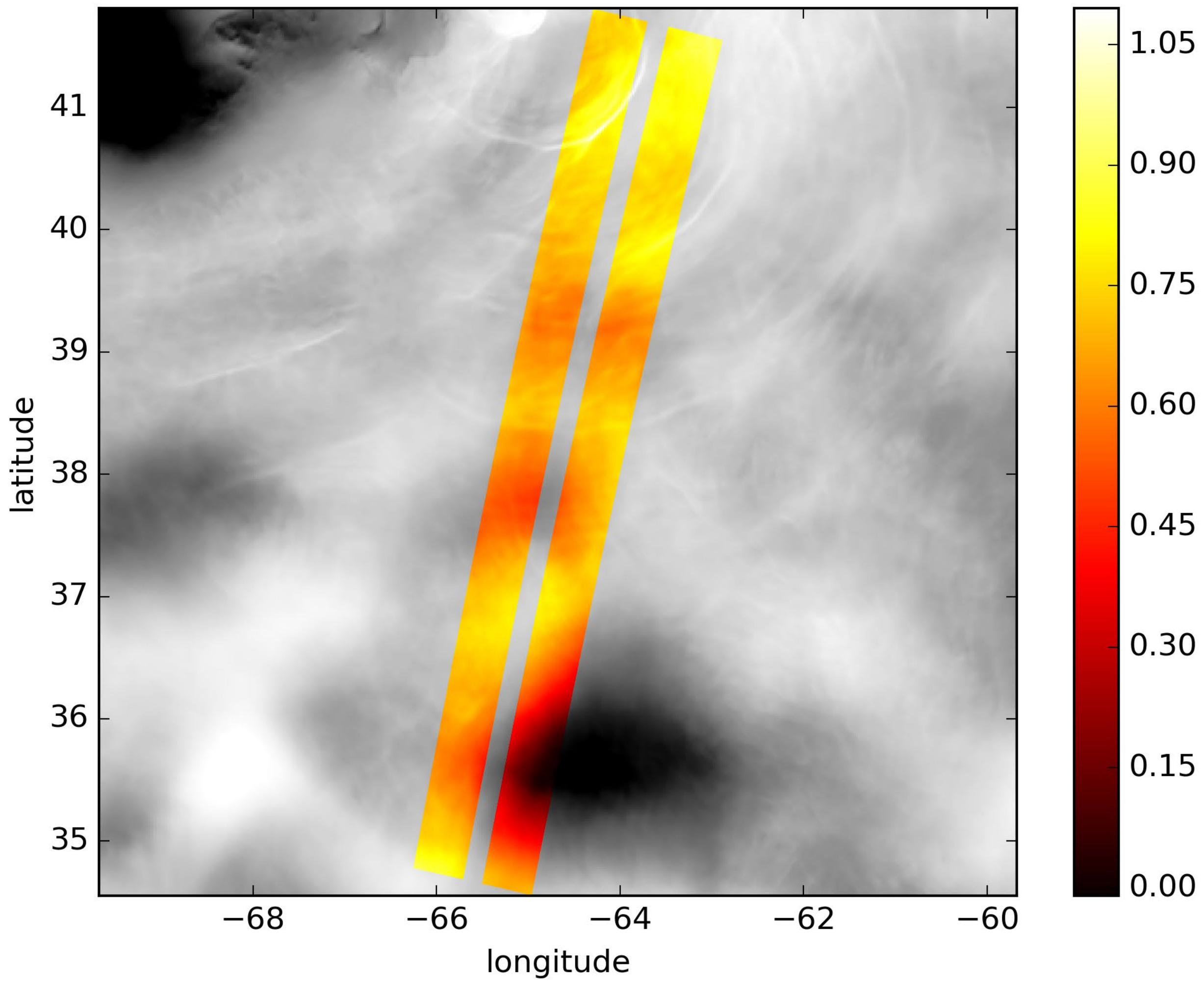
$$u = \frac{k\omega + ilf}{\omega^2 - f^2}\frac{p'}{\rho_0} \qquad v = \frac{l\omega - ikf}{\omega^2 - f^2}\frac{p'}{\rho_0}$$

$$\rho' = \rho_0 \frac{-imN^2}{g(N^2 - \omega^2)}\frac{p'}{\rho_0}$$

$homogeneous\ solution\ (interior\ mode)$
$+\ particular\ solution\ (SQG)$

$$\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial F_m}{\partial z}\right) = -R_m^{-2}F_m; \quad \frac{dF_m}{dz} = 0 \quad \text{at} \quad z = 0, -H.$$

A challenge in using SWOT SSH



A snapshot of SSH from the 1/48th MITgcm

interior+surface Quasi-geostrophy+Waves (**isQGW**)

Based on Wang et al. (2013) and Ponte et al. (2017)

Interior QG

$$\eta = \eta^{sg} + \eta^{ig} + \eta^{iw}$$

SQG

waves

Share the QG dynamics

Share the baroclinic normal mode basis

η^{ig} is a problem

Wang et al. (2013): $\eta^{ig} = \eta - \eta^{sg}$

Does not distinguish η^{ig} and η^{iw}

Ponte et al. (2017):

$\eta^{iw} = \eta - \eta^{sg}(eSQG)$

Does not consider the vertical PV tilt.

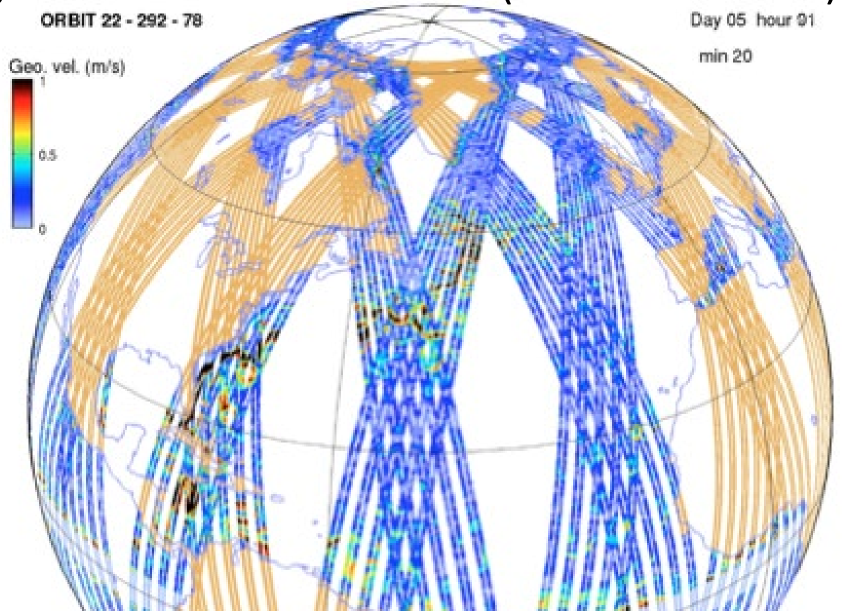
Challenges in using SWOT SSH

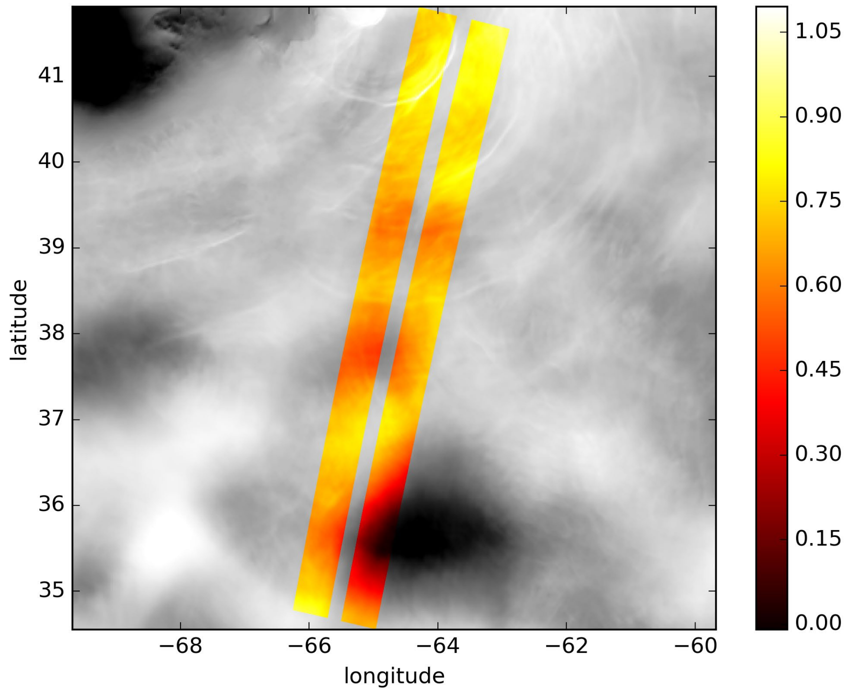
1. Gaps (both in space and in time)
2. Coexistence of internal waves and low eddies coexist in SWOT SSH

Two objectives

1. Fill the gaps
2. Separate waves and eddies from snapshot SSHs, with the aid of other available data

Image credit: SWOT simulator (Ubelmann et al.)





A snapshot of the steric height (1/48th MITgcm)