Moored time series and frequency-wavenumber spectra

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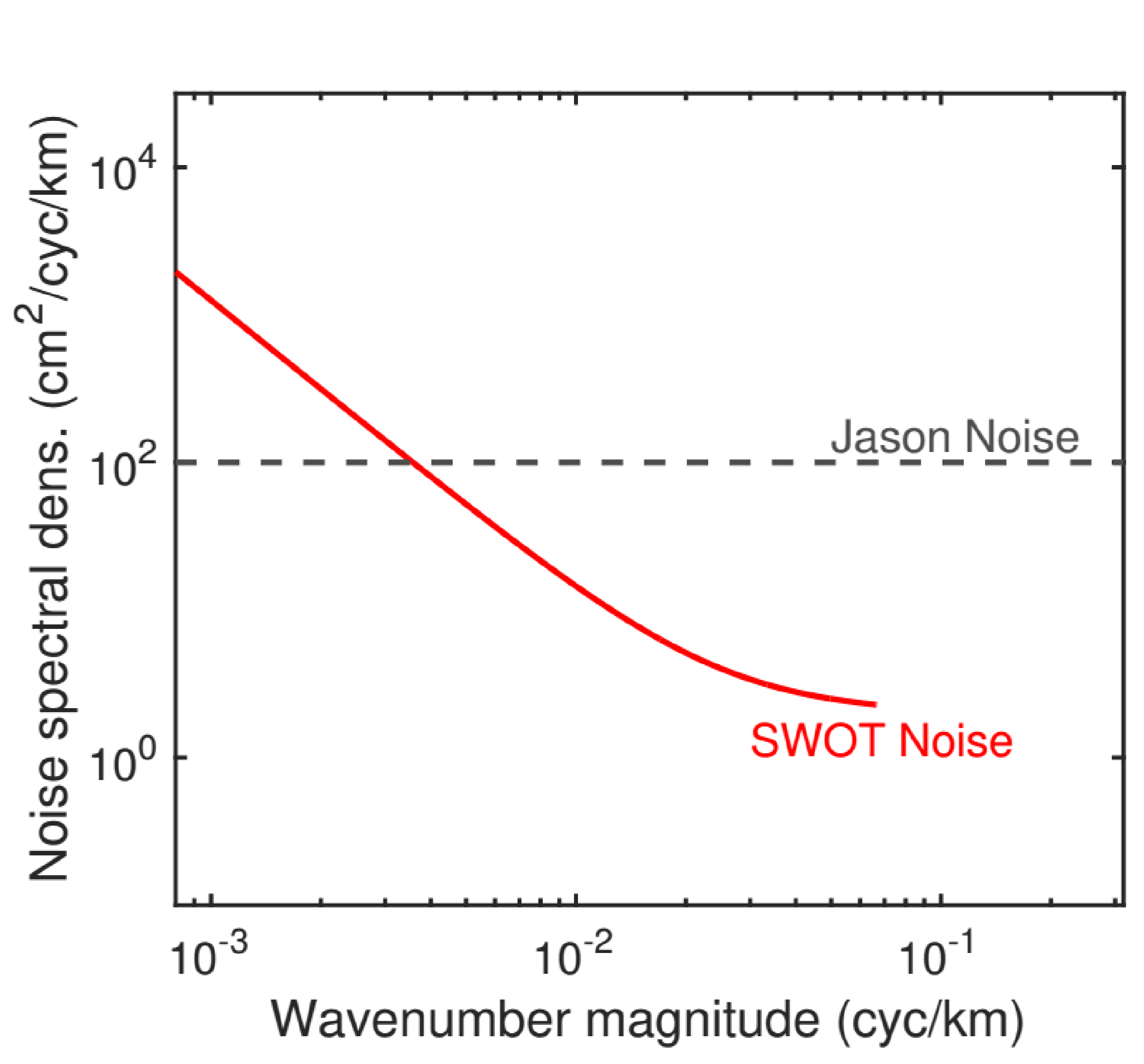
SWOT will reveal an exciting new view of 10-100km SSH variability

We need a better understanding of the SSH variability at these scales.

We expect internal waves to have measurable SSH signals.



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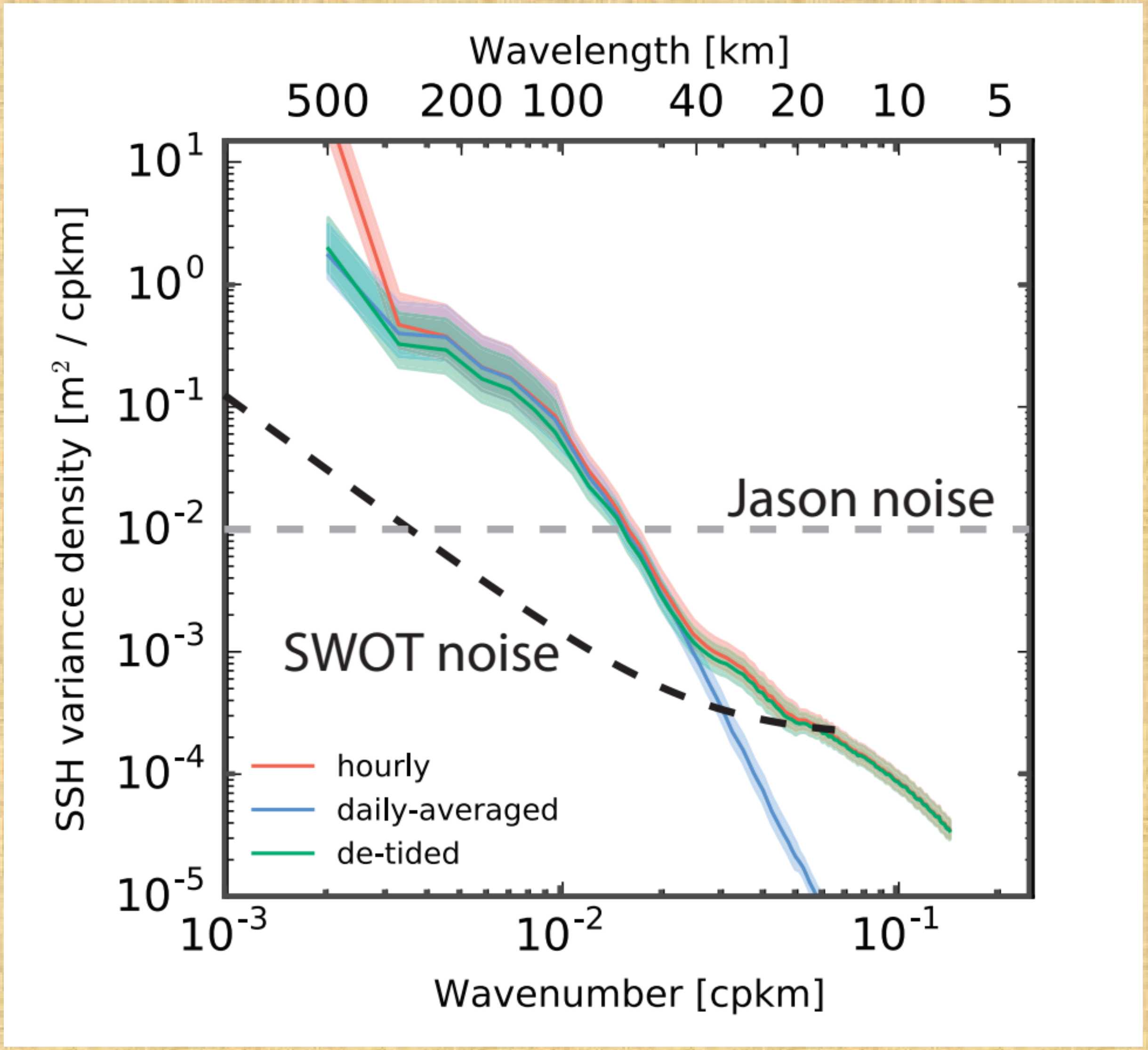
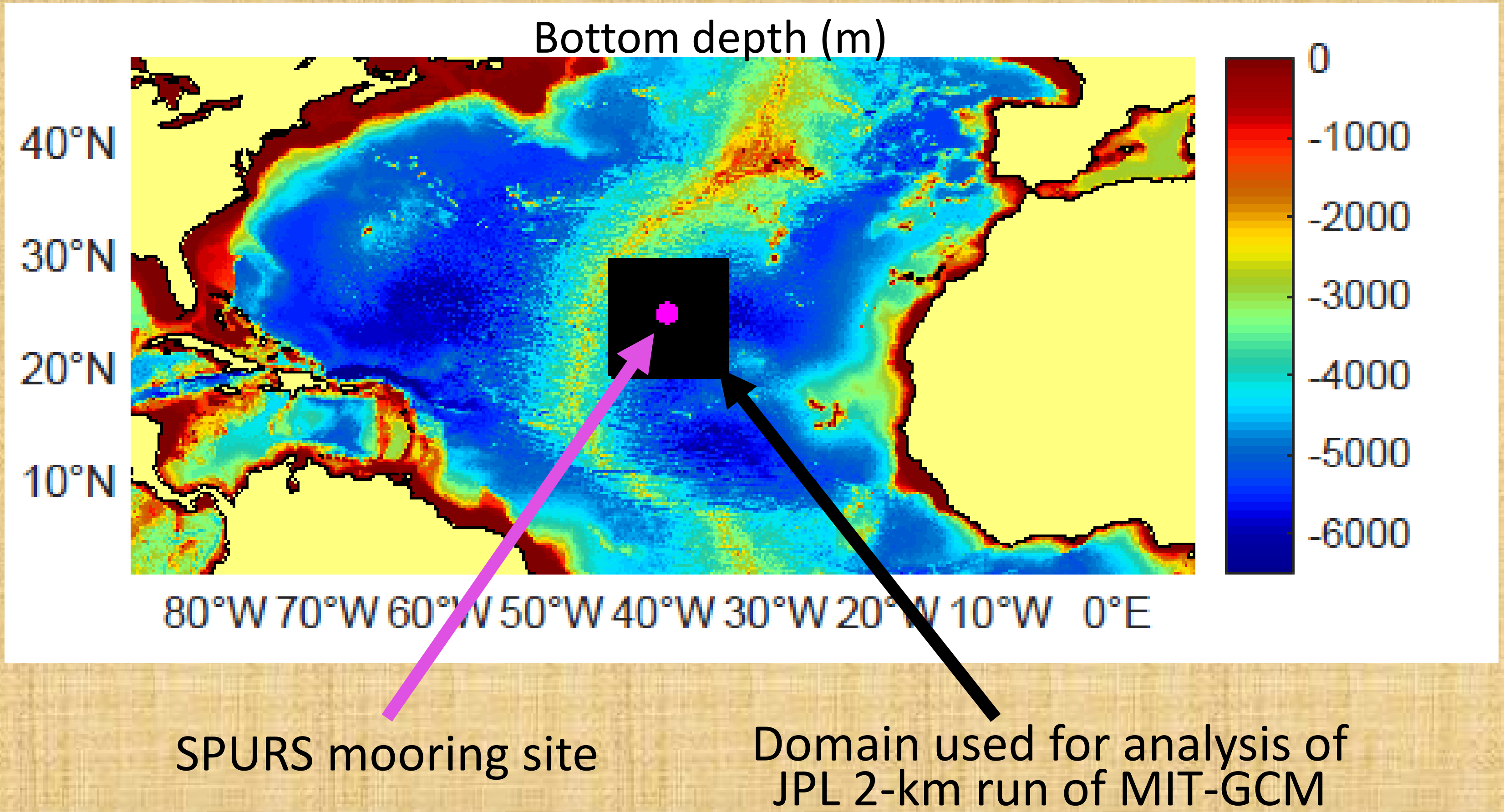


Morrow et al. (2017); modified after Rocha et al. (2016)

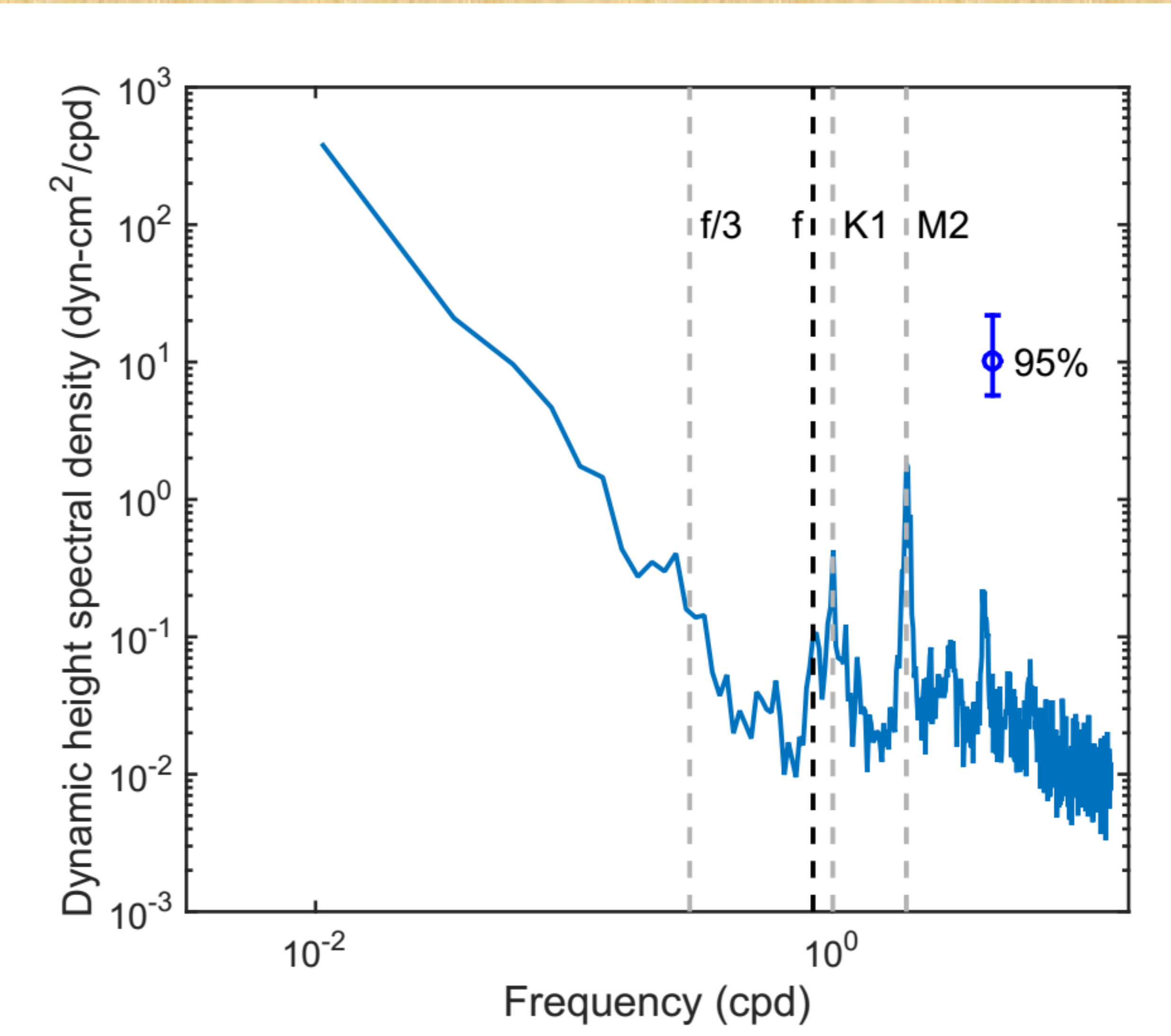
We can use mooring data and GCMs to estimate the SSH wavenumber spectrum, subject to numerous assumptions



We had a surface mooring in SPURS measuring temperature and salinity with very good temporal and vertical resolution in the upper 400m (Farrar et al., 2015)

Using mooring data to estimate the SSH wavenumber spectrum of internal waves

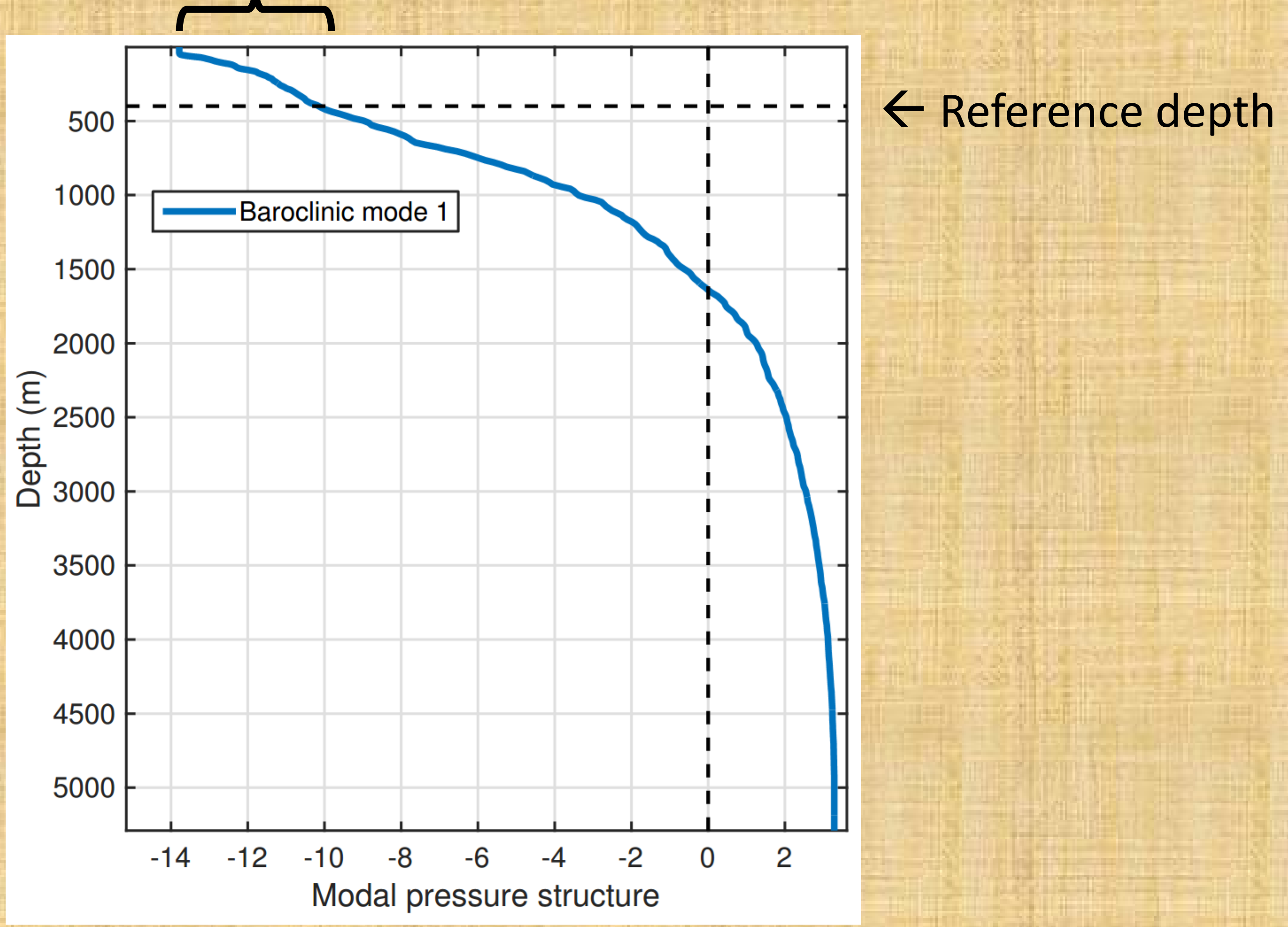
Frequency spectrum of surface dynamic height relative to 400m from the SPURS mooring



Using mooring data to estimate the SSH wavenumber spectrum of internal waves

Dynamic height represents the dynamically driven pressure difference over some depth range

Pressure difference between surface and reference depth



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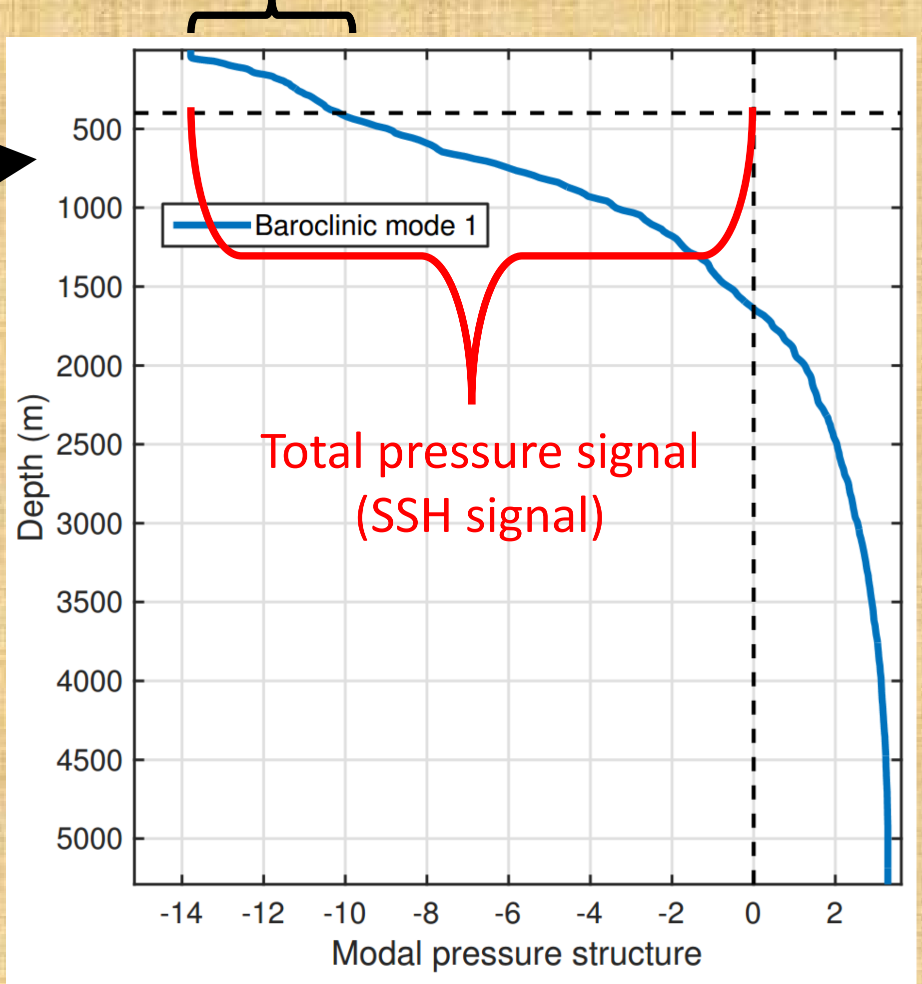
Relationship of dynamic height to SSH signal of baroclinic motions

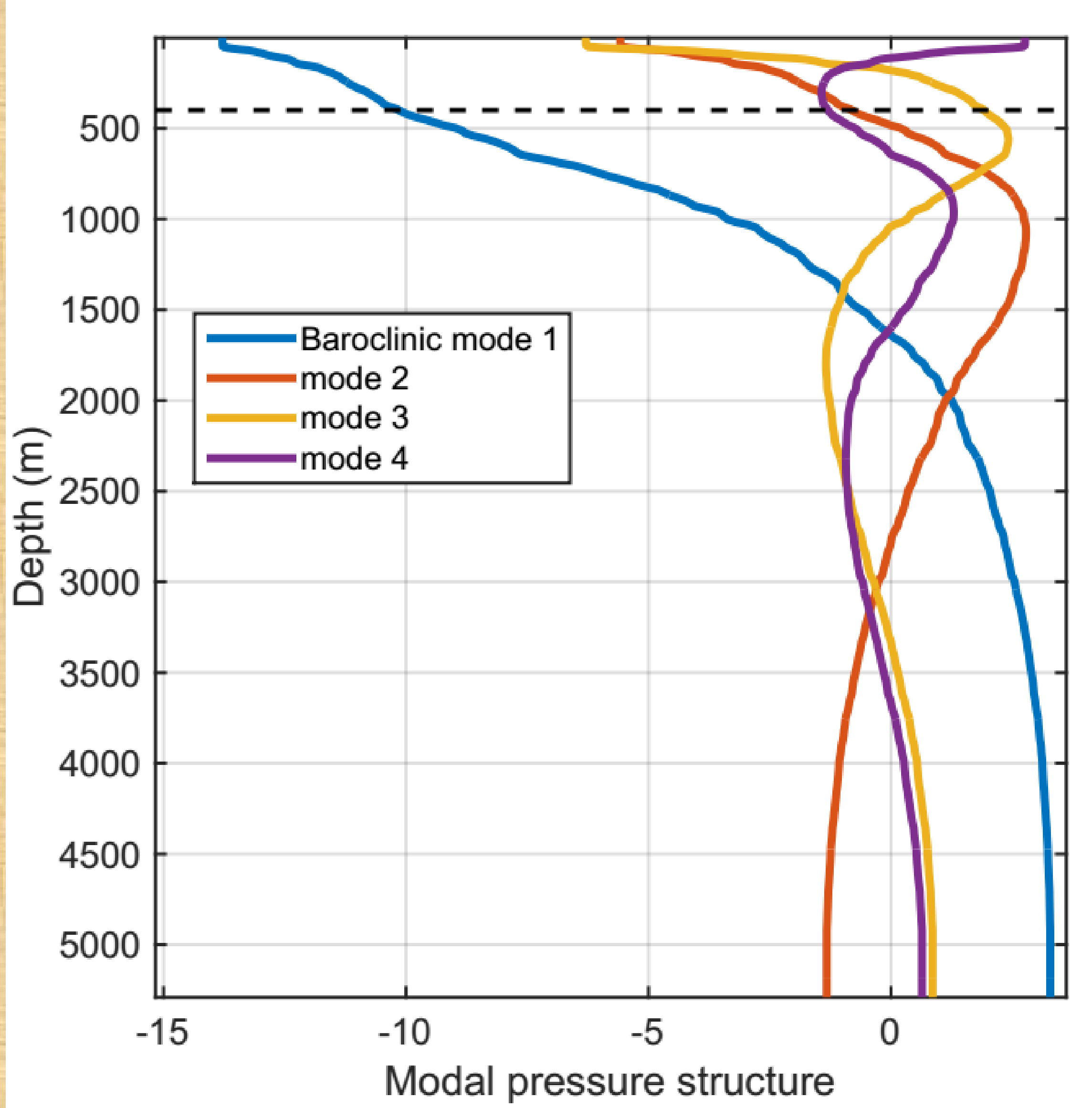
Pressure difference between surface and reference depth (dynamic height)



← Reference depth

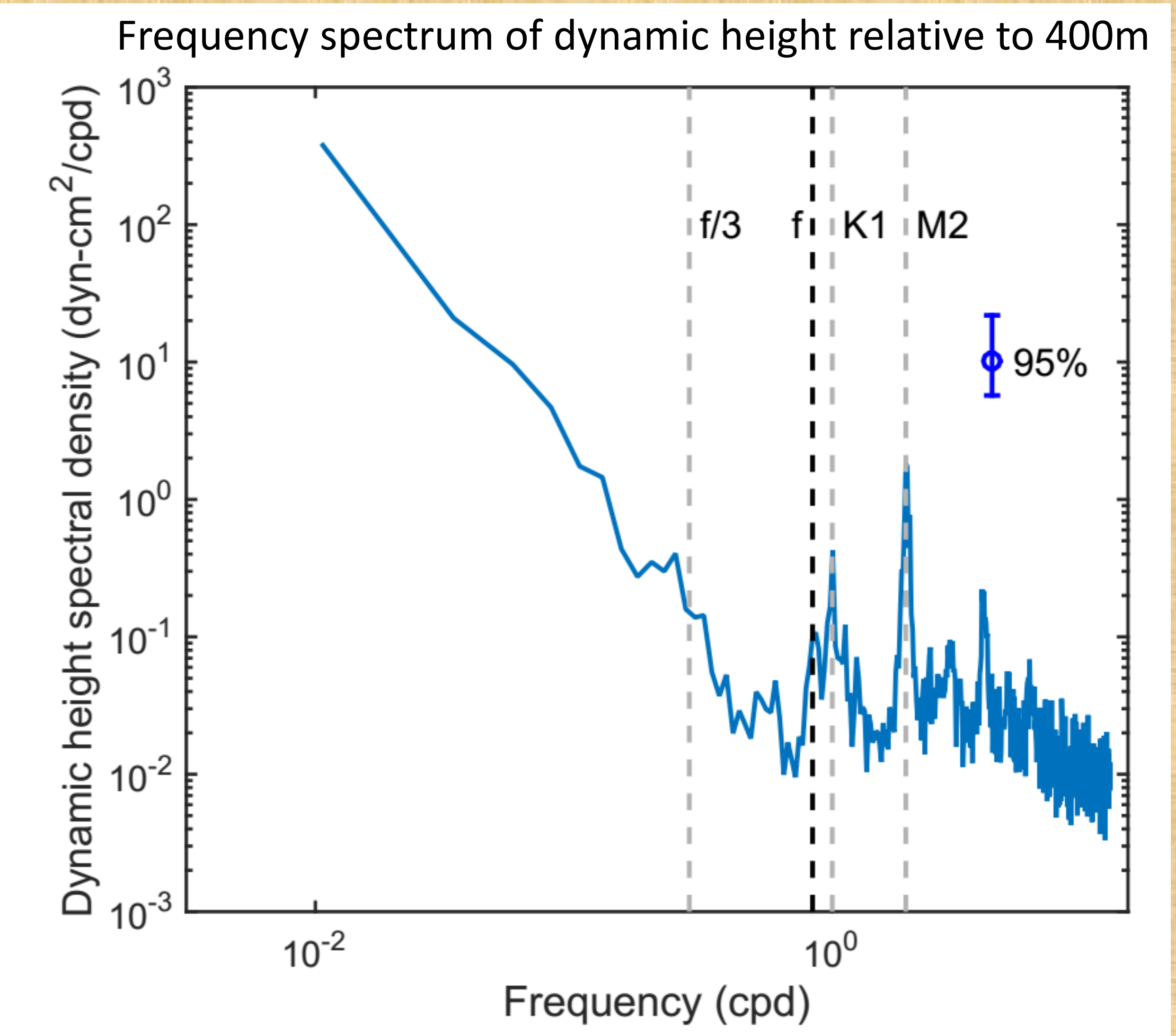
Using mooring data to estimate the SSH wavenumber spectrum of internal waves

If we know how the dynamic height or SSH variance is partitioned between vertical modes, then it is easy to relate the two.



Using mooring data to estimate the SSH wavenumber spectrum of internal waves

If we know how the dynamic height or SSH variance is partitioned between vertical modes, we can also convert between frequency and wavenumber using IW theory



Using mooring data to estimate the SSH wavenumber spectrum of internal waves

We can convert a frequency spectrum, $\tilde{\Psi}(\omega)$, to a horizontal wavenumber spectrum, $\Psi(k)$, given a dispersion relation $\omega(k_h)$:

$$\Psi(k_n) = \frac{d\omega_n}{dk_n}\tilde{\Psi}(\omega_n)$$

For internal waves:

$$\omega_n{}^2 = c_n{}^2(k_n^2) + f^2$$

$$\frac{d\omega_n}{dk_n} = \frac{c_n{}^2}{\omega_n}\left(\frac{\omega_n{}^2 - f^2}{c_n{}^2}\right)^{\frac{1}{2}}$$

(c_n is a constant, different for each baroclinic mode)

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This frequency-wavenumber transformation should hold for each mode, but the difficulty is to determine the relative strength of different modes

Using mooring data to estimate the SSH wavenumber spectrum of internal waves

If the vertical modes are not phase locked, the SSH signal associated with mode *n* is:

$$\Psi_n^{SSH}(k_h) = a_n \Psi_n^{dyn}(k_h)$$

(a_n is a known constant relating dynamic height amplitude to SSH amplitude for each baroclinic mode; $a_1=4a_2=25a_3=30a_4$)

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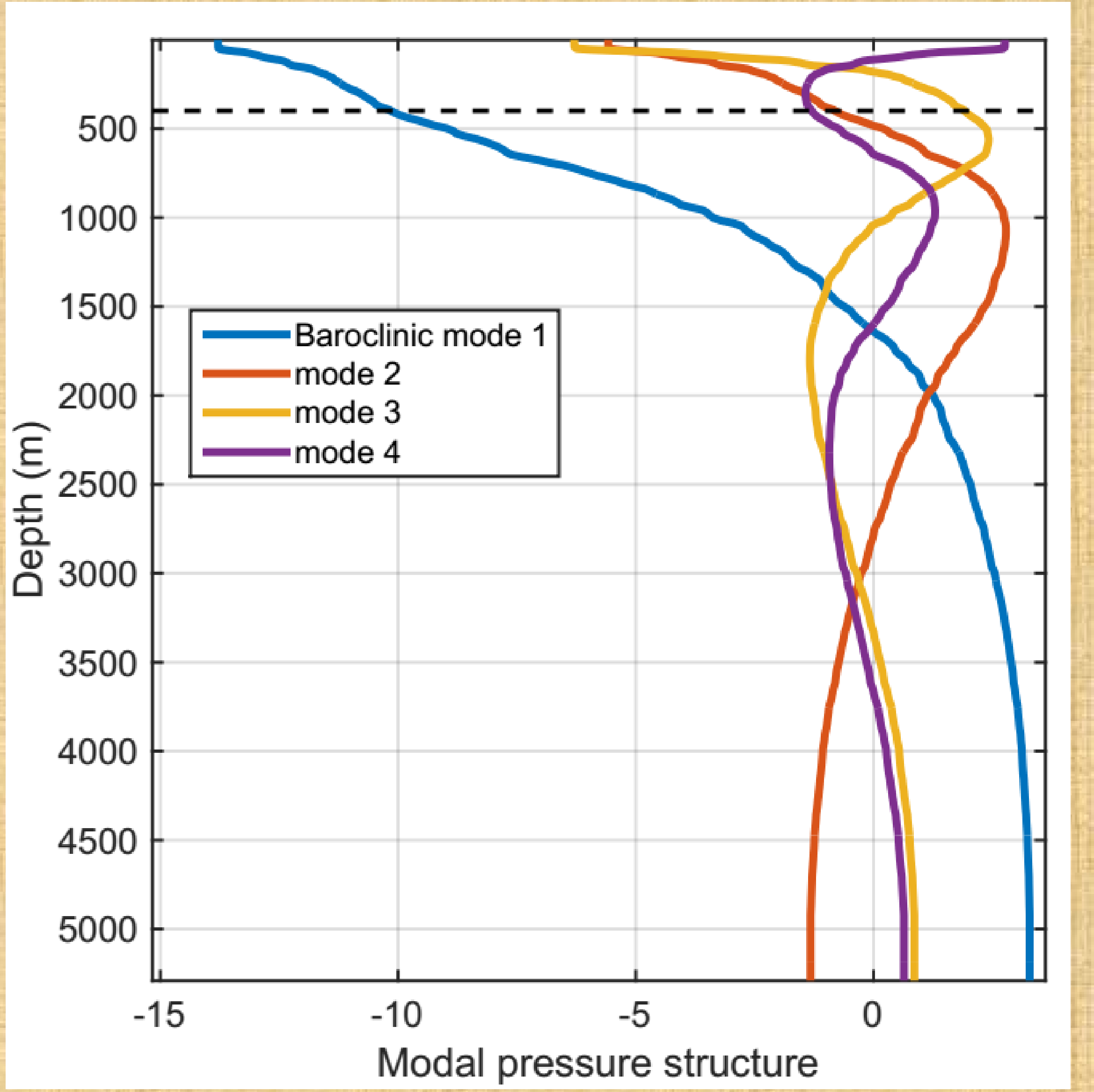

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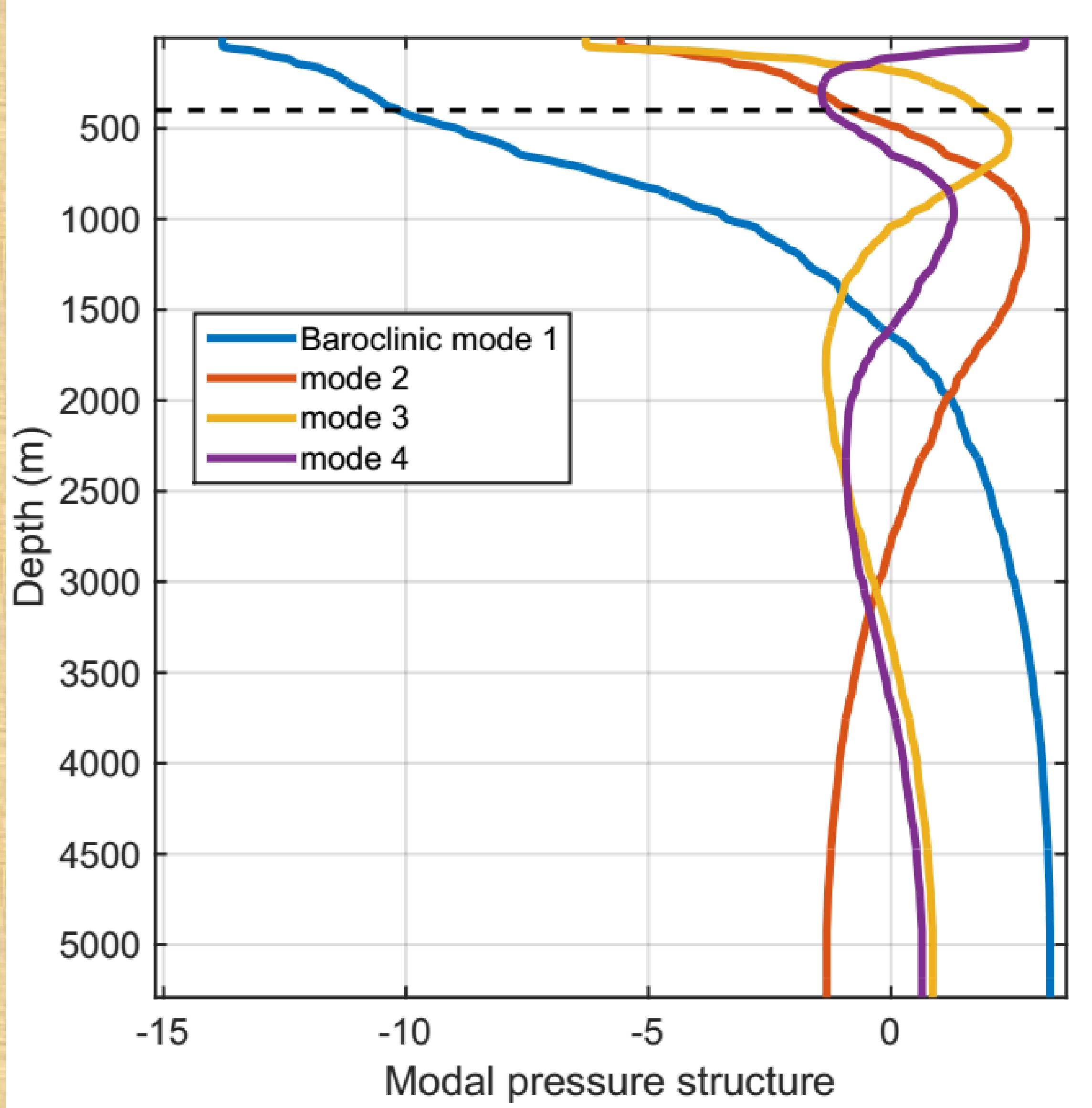
Suppose the lowest 4 modes contribute equally to dynamic height variance (25% mode 1)



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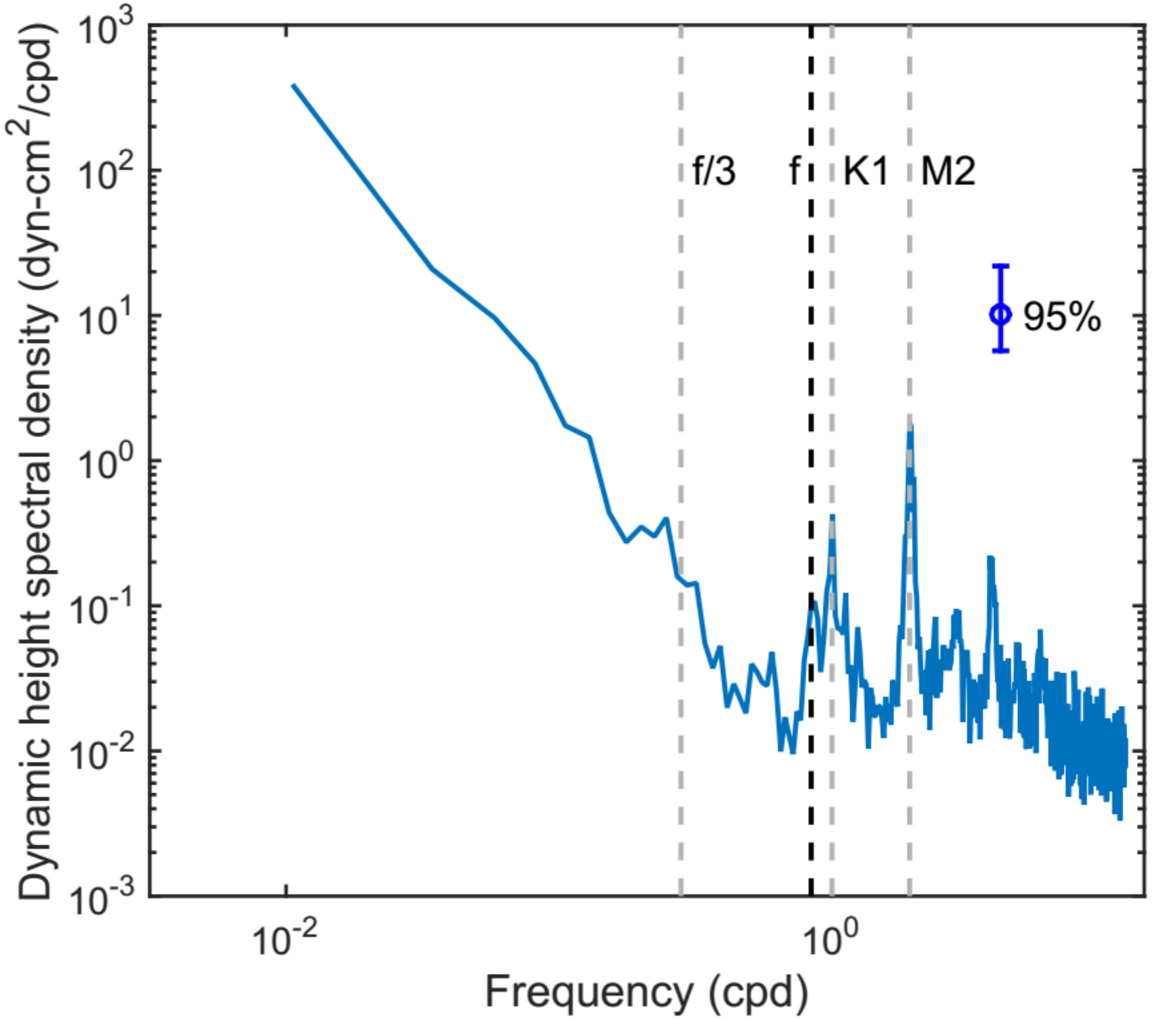
Frequency spectrum of dynamic height relative to 400m



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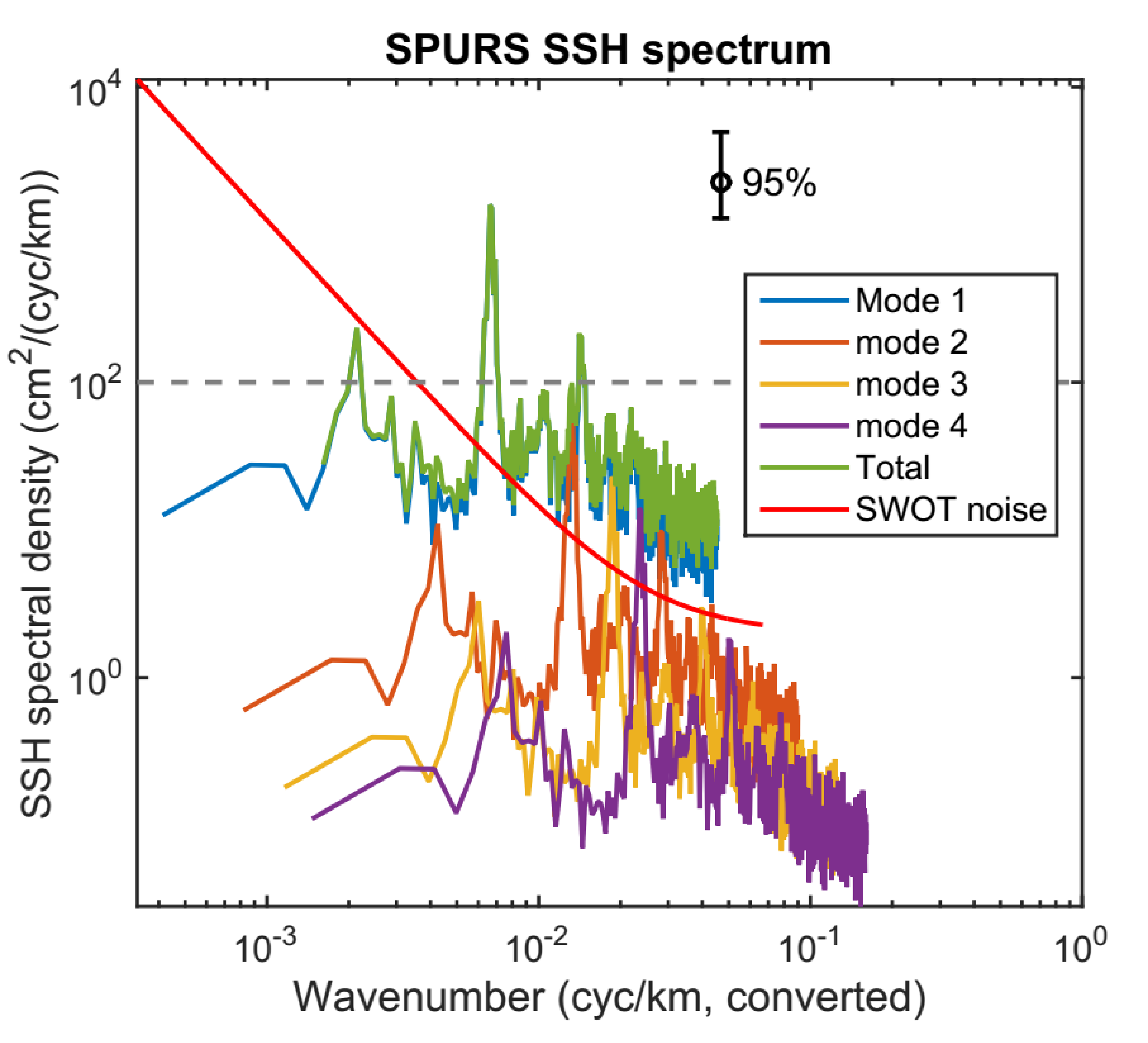

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This error bar is a factor of 3.8; if mode 1 instead contributes 95%, the estimate would be the same within error bar

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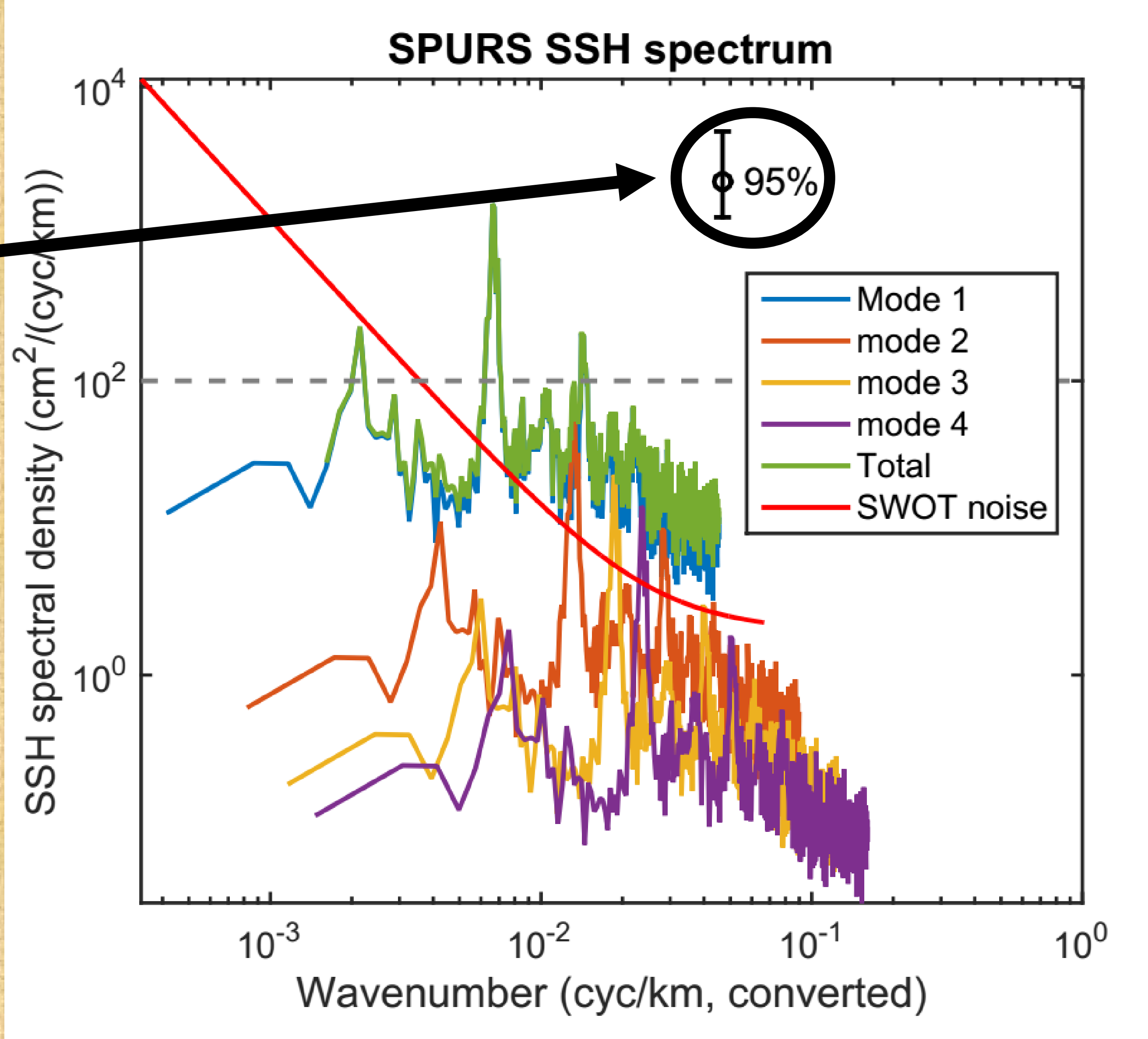
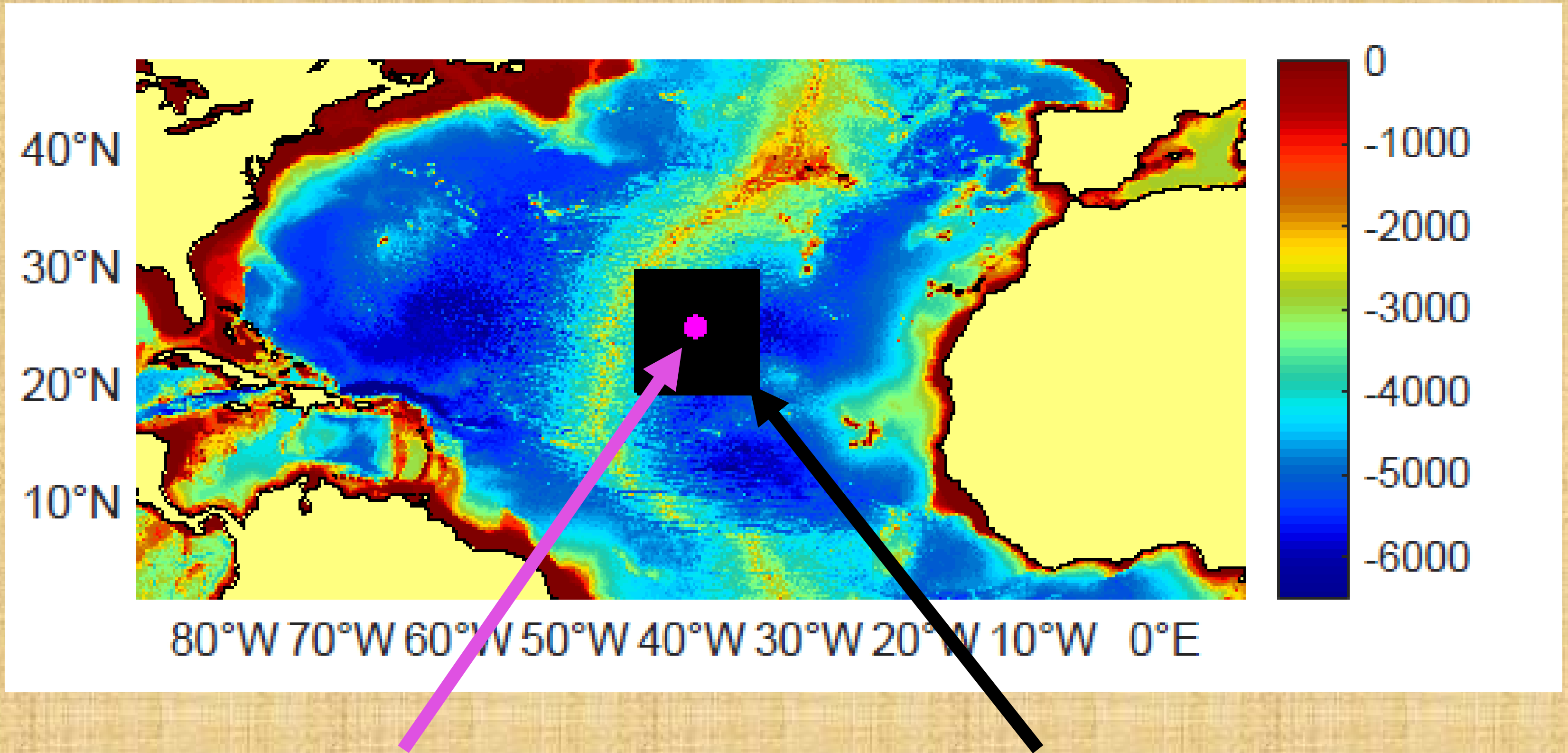


With new GCM runs, we can directly the SSH wavenumber-frequency spectrum of internal waves



SPURS mooring site

Domain used for analysis of JPL 2-km run of MIT-GCM

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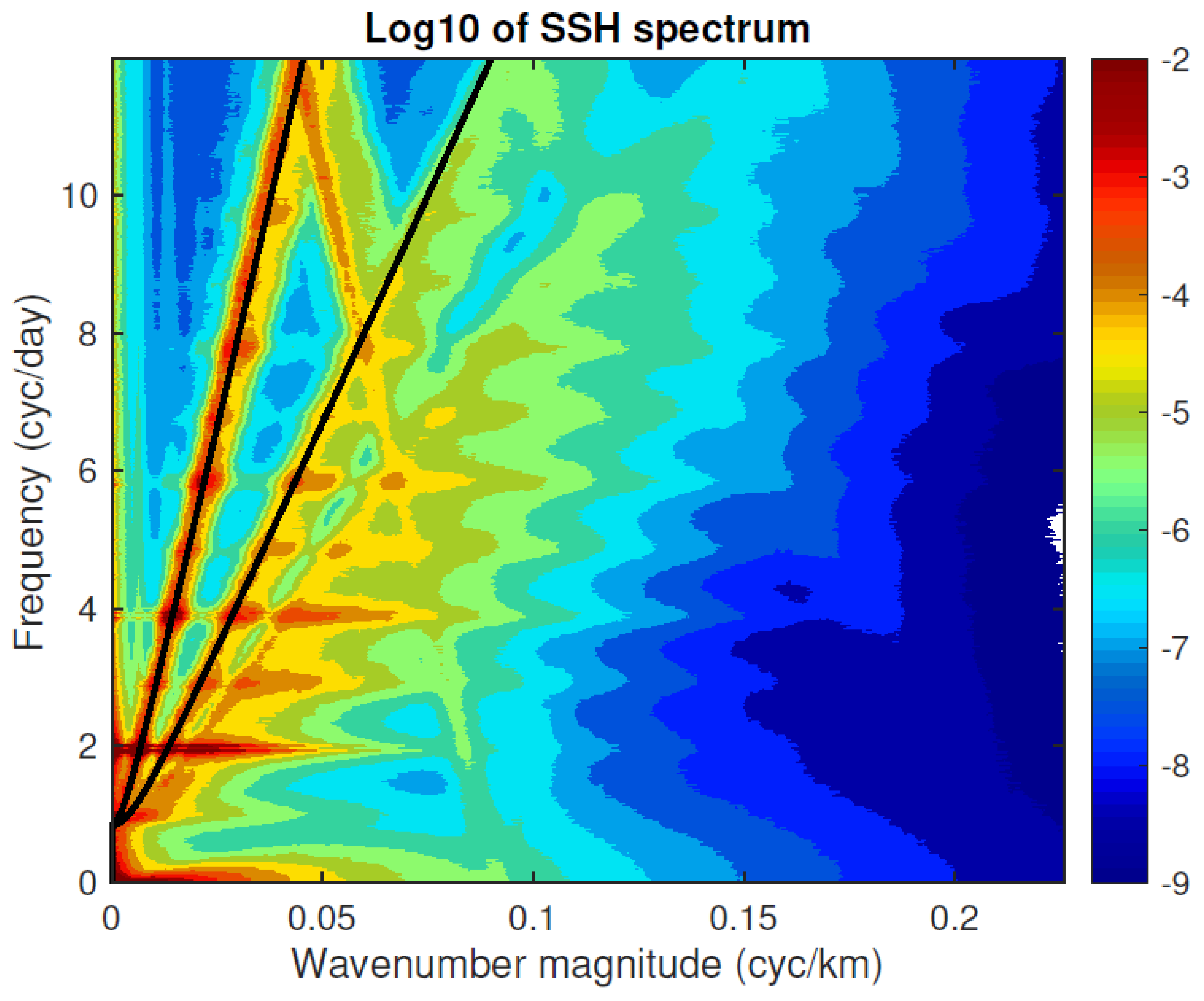

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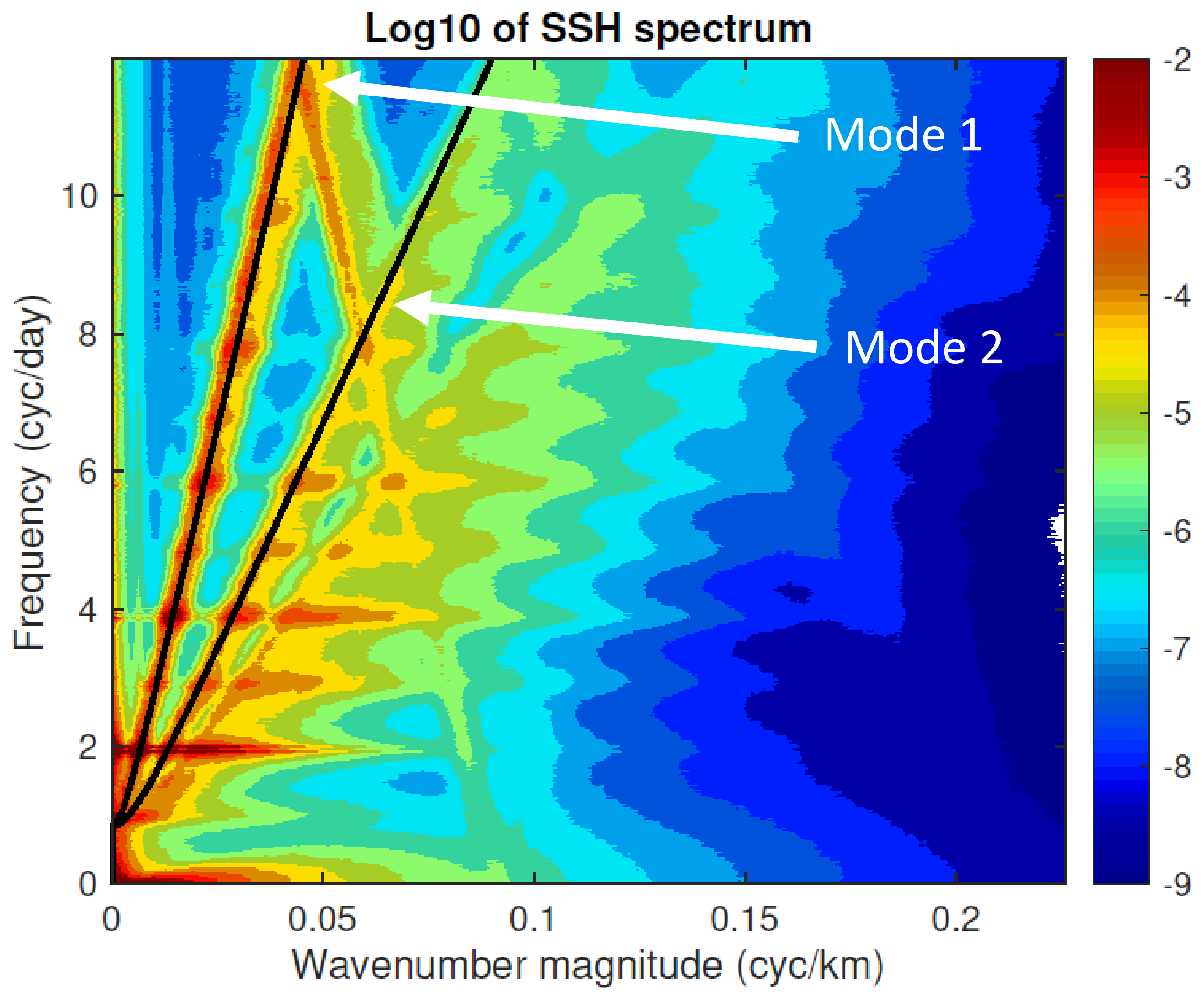

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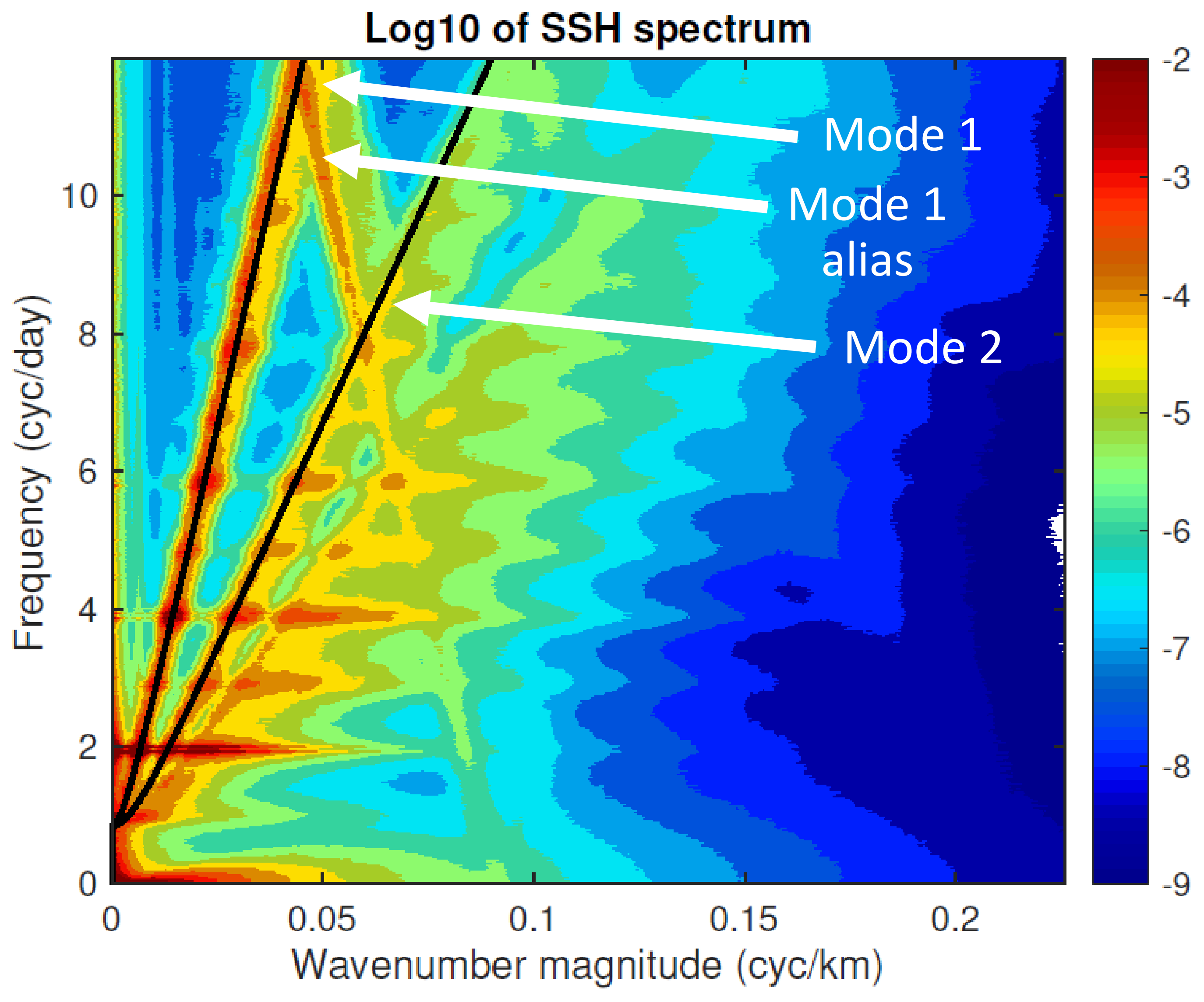

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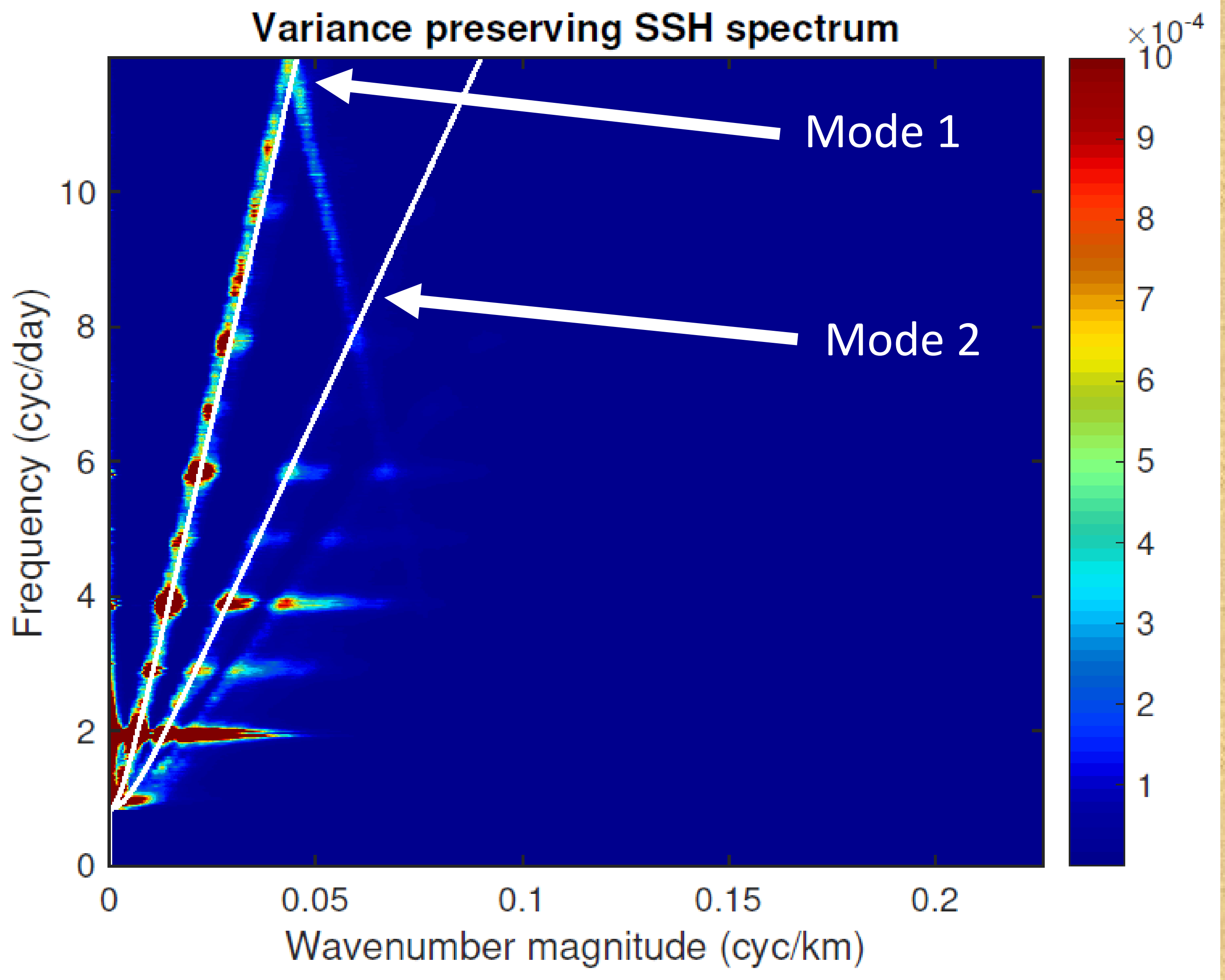

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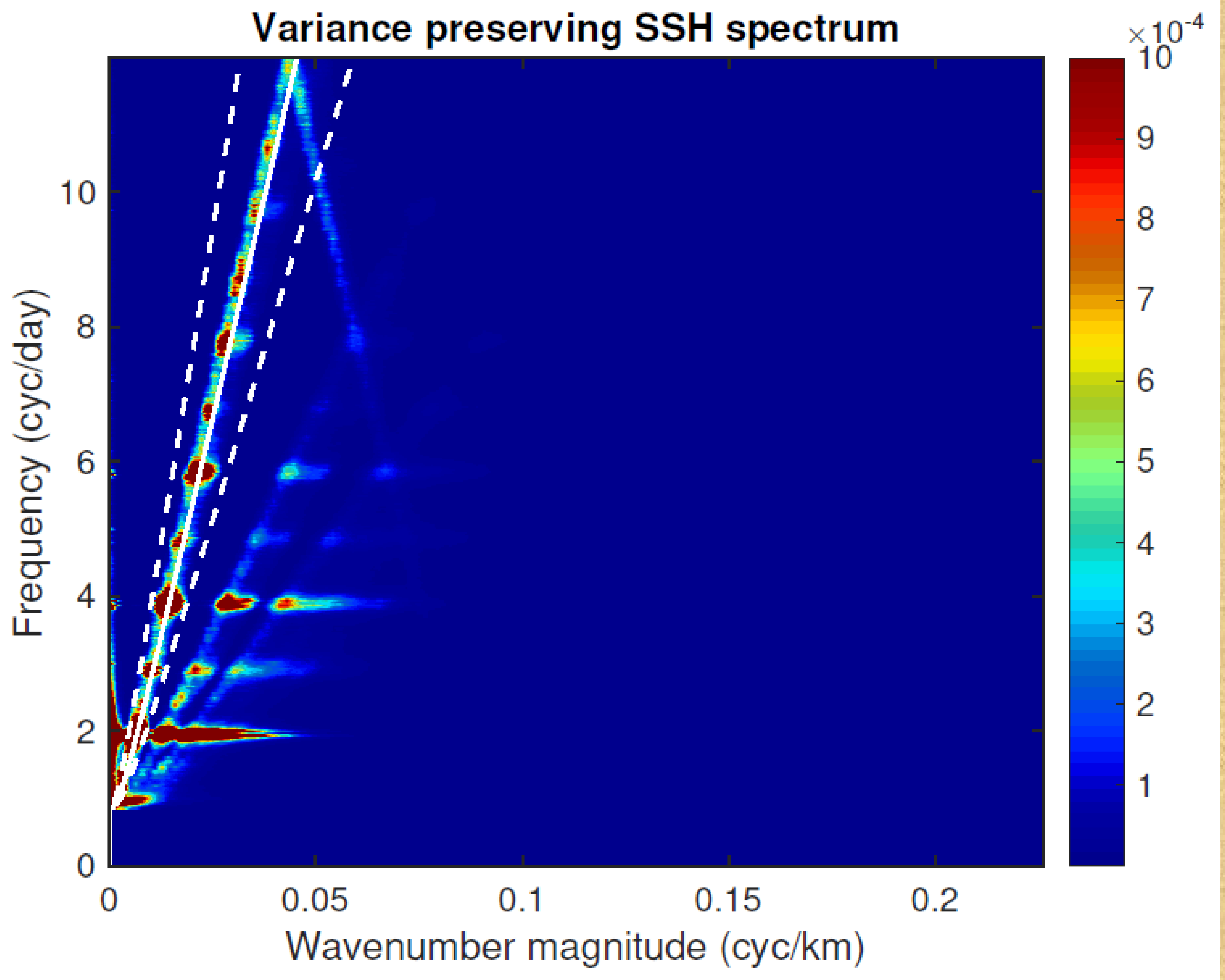
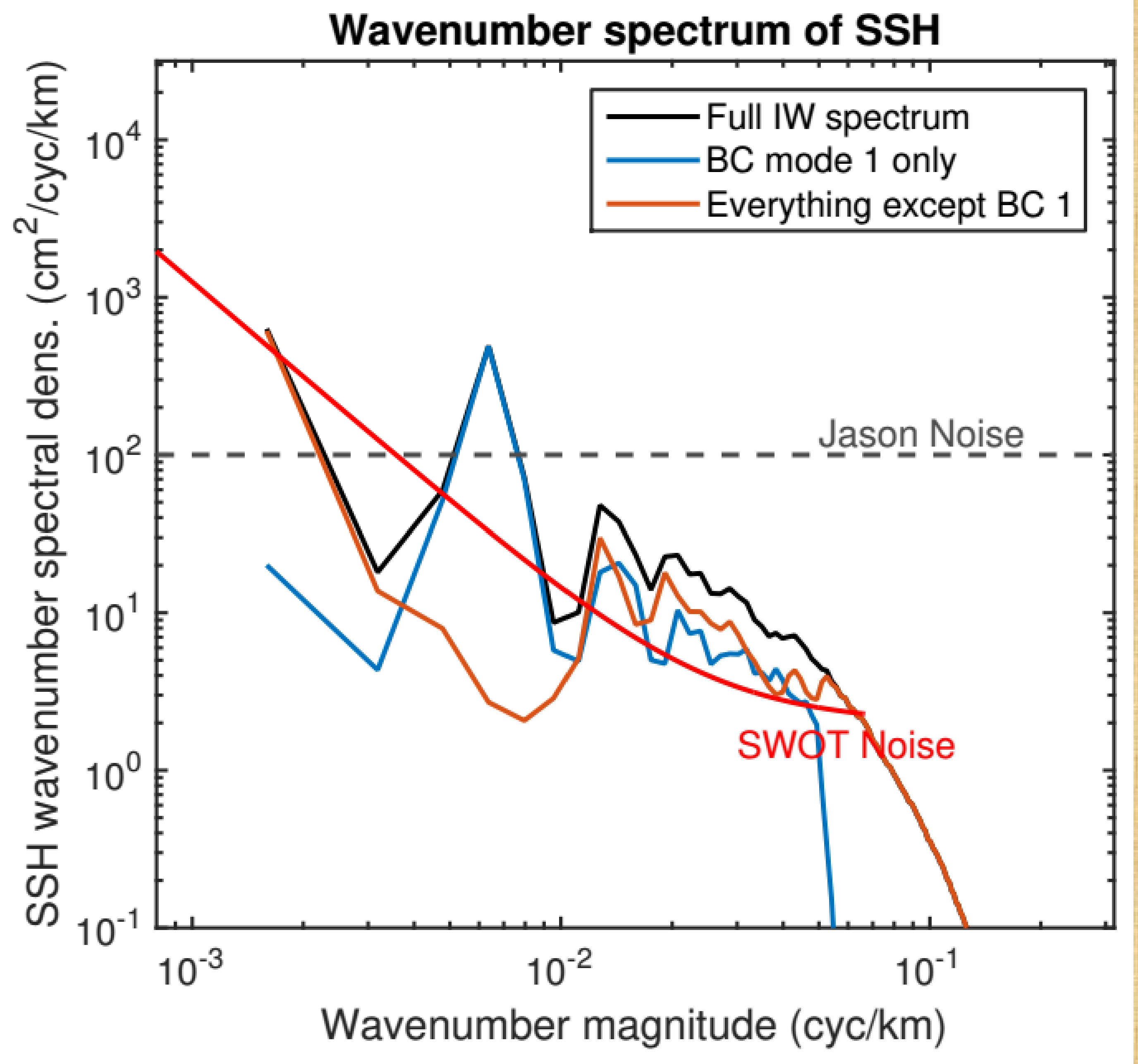
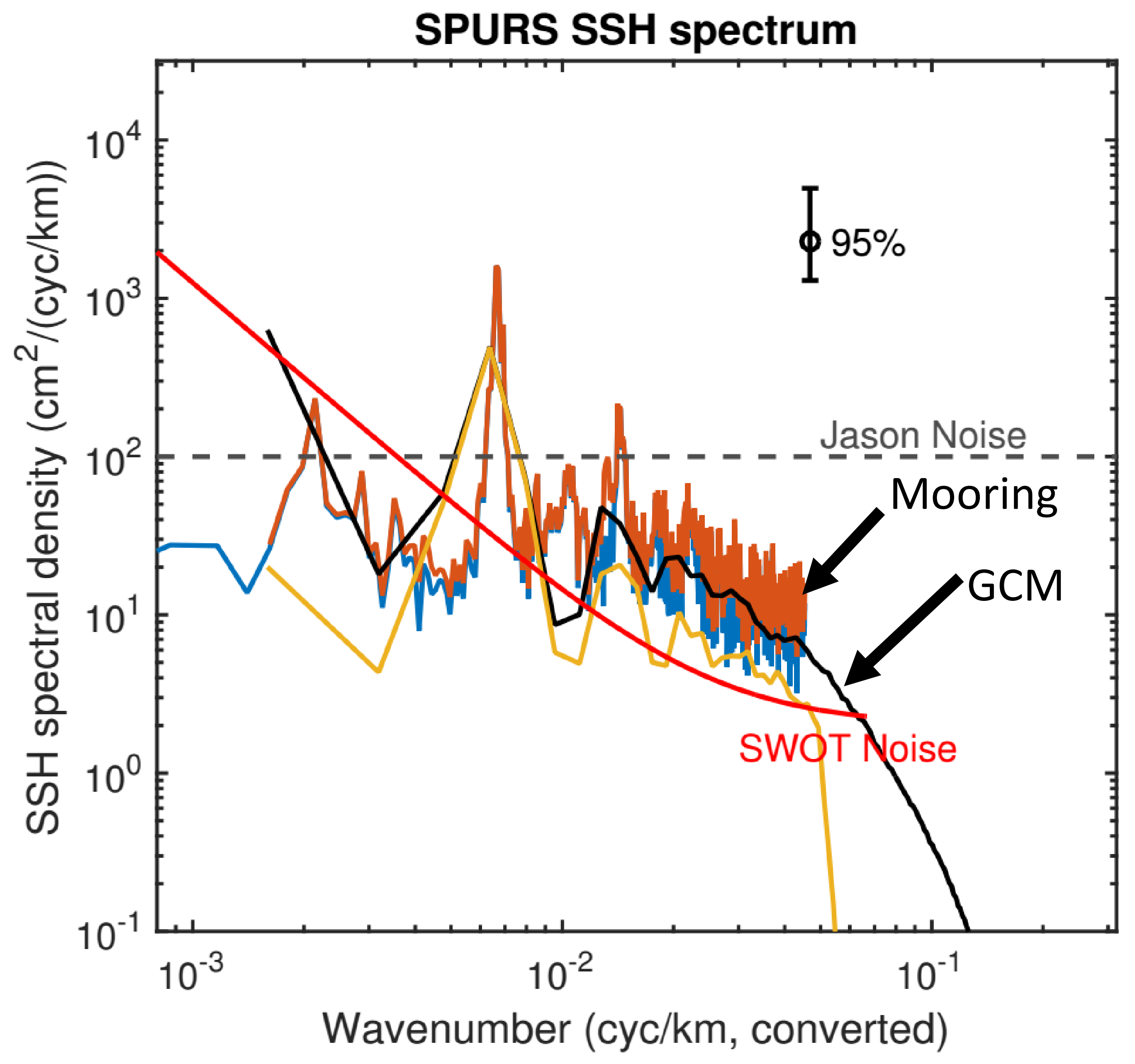
With the model, we can separate mode 1 contribution to SSH spectrum





Wavenumber spectrum of SSH
Full IW spectrum
BC mode 1 only
Everything except BC 1
Jason Noise
SWOT Noise
SSH wavenumber spectral dens. (cm^2/cyc/km)
Wavenumber magnitude (cyc/km)
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-3}
10^{-2}
10^{-1}

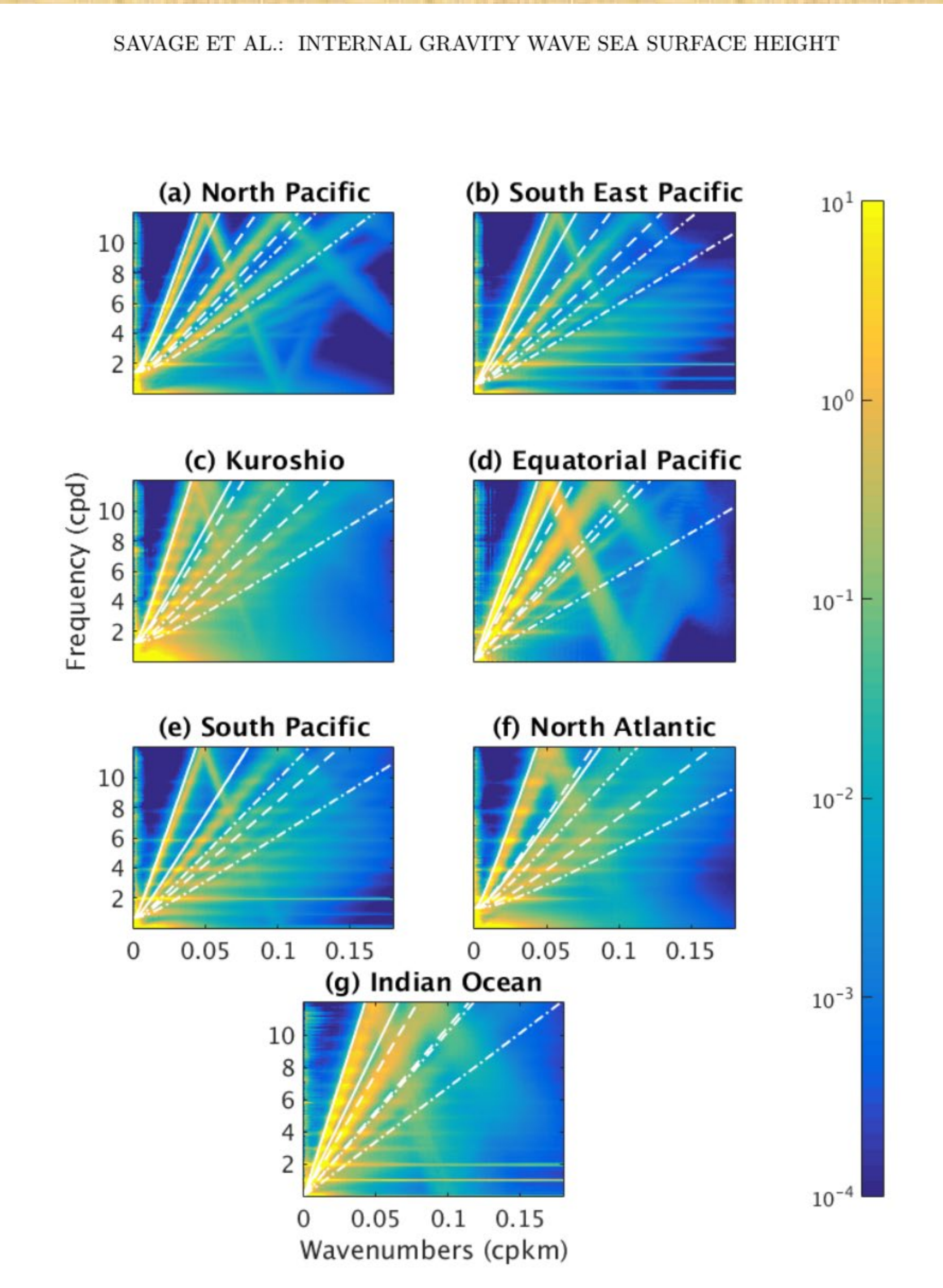
The model and mooring estimates are roughly consistent



Conclusions

1. We would like to know the rough magnitude of IW signal (time-space scales of SSH) for planning SWOT cal/val and data products
2. The mooring estimate is most sensitive to assumed portion of dynamic height signal in mode 1 (~linearly proportional)
3. Both mooring and GCM estimates of SSH wavenumber spectrum are uncertain by an order of magnitude
4. SWOT will provide much more information on internal wave SSH than is now available

Different regions, same model (MITgcm, 2km run)



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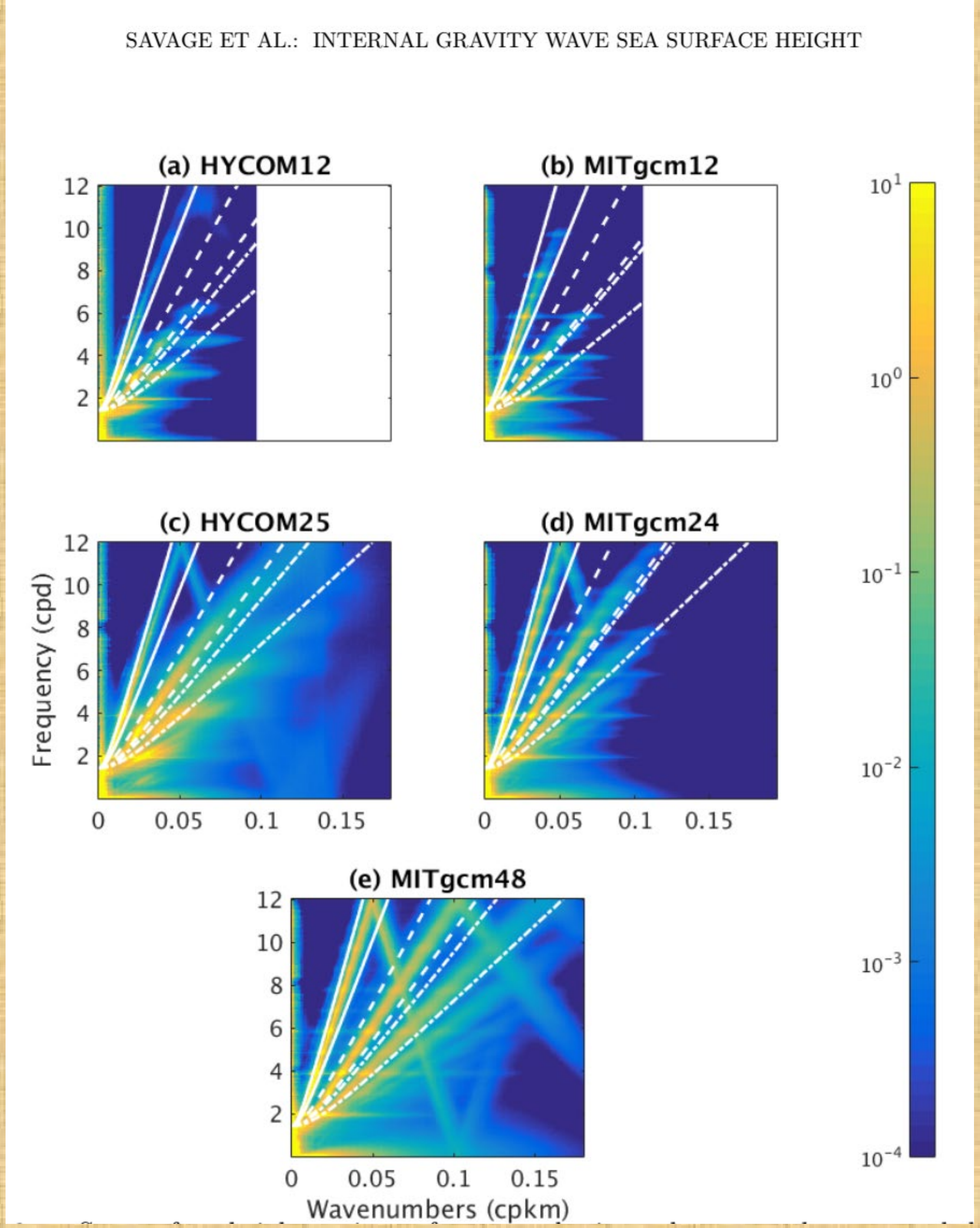

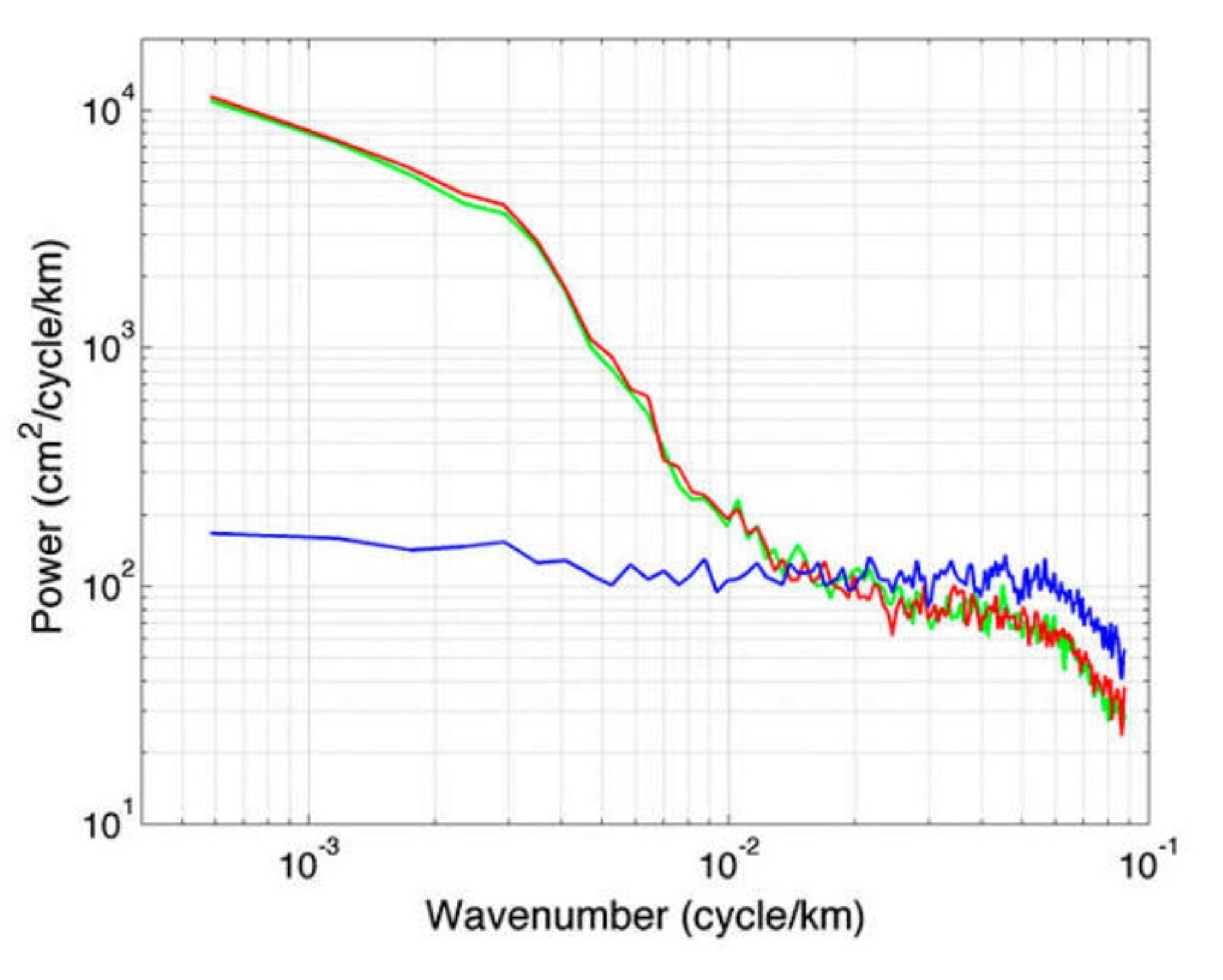

Figure 9. Along-track SSH wavenumber spectra of *Jason-1* (green line) and *Jason-2* (red line) along a pass in the eastern Pacific during a time when the two satellites sampled the same ground track with a separation of 1 minute. The blue line shows the spectrum of their difference.