Physical-based inverse methods

to infer ice-sheets & ice-streams bed properties

by J. Monnier

INSA & Mathematics Institute of Toulouse

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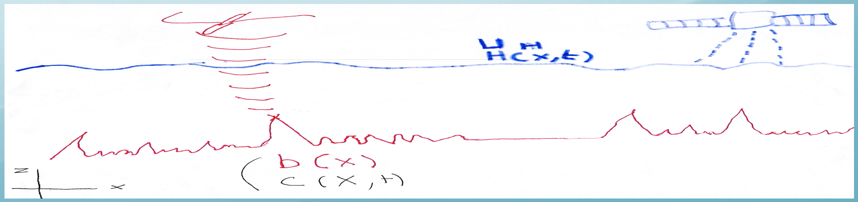
Study #1) Ice-sheet beds in interior sectors :

➔ A new inverse method to improve the bed estimations ?

Study #2) On multi-scale dynamic inversions
to be applied to seasonal ice-streams modeling



Antarctica Bedmap 2: Topography from data compilation & interpolation

Source : [Fretwell et al.]'13

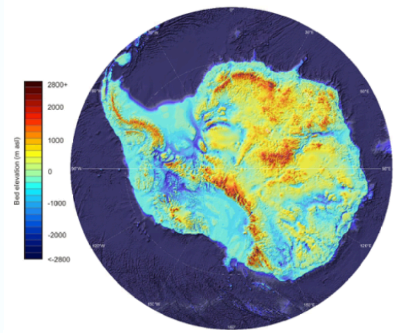
Method: airborne data compilation, 25 millions of survey points
+ Kriging & other.

➔5-km cells.

1/3 of cells contain data ➔ Error +/-100m

The others 2/3 do not ➔ Error +/- 300m

80% of cells are within 20 km of measurements

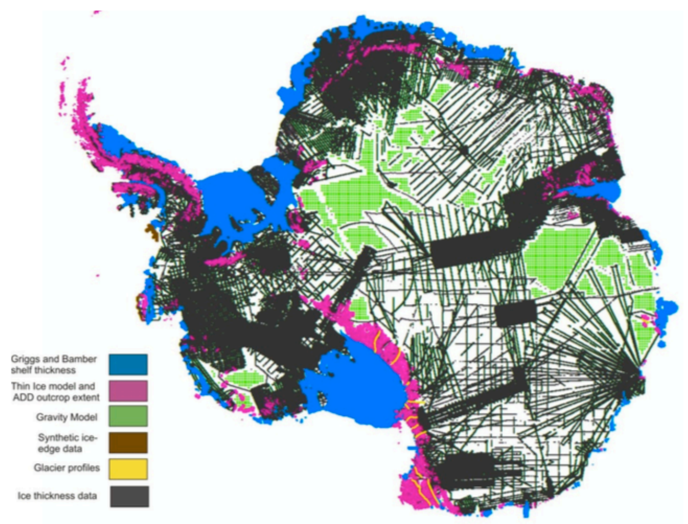


Bedmap2: bed elevation
[Fretwell et al.]'13

For interior sectors, gravity data are inverted ➔ **Error up to +/-1000 m**

« Poles of ignorance » are ~200Km from nearest data.

Data set used for Bedmap2



Uncertainties

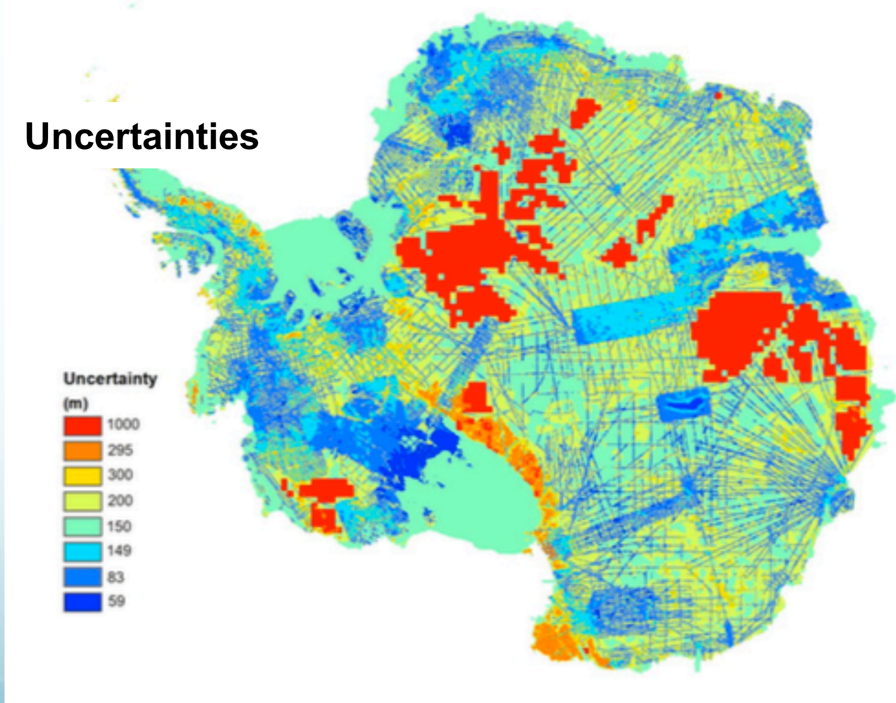


Fig. 11. Estimated uncertainty in ice thickness grid.

Greenland bed topography from data compilation & interpolation

Source : [Bamber et al.]'13

From airborne data, 420 000 km of lines : see Fig.
Data sources: CReSIS & many others.

Method: data compilation (data gridded with 5km posting)
+ Averaging & Kriging on 1km - 2.5km grids.
➔ **Errors** from 10 **to 300 m**.

For interior sectors,
across-track spacing can be more than 50 km
➔ **Error +/-100 m and up to +/-300m.**

Challenge for physical-based inverse methods:
Improve the accuracy
between the data lines
by combining
the surface observations
(Altimetry, InSAR)
with
the conservation laws

RMSE

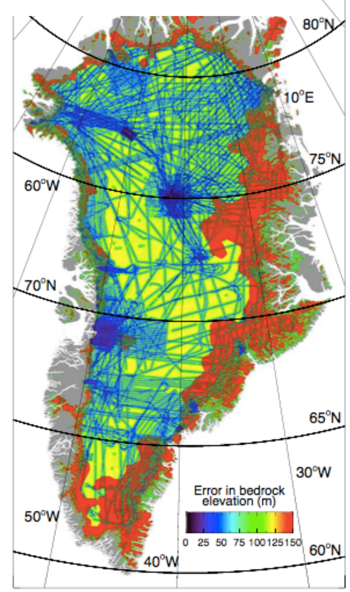
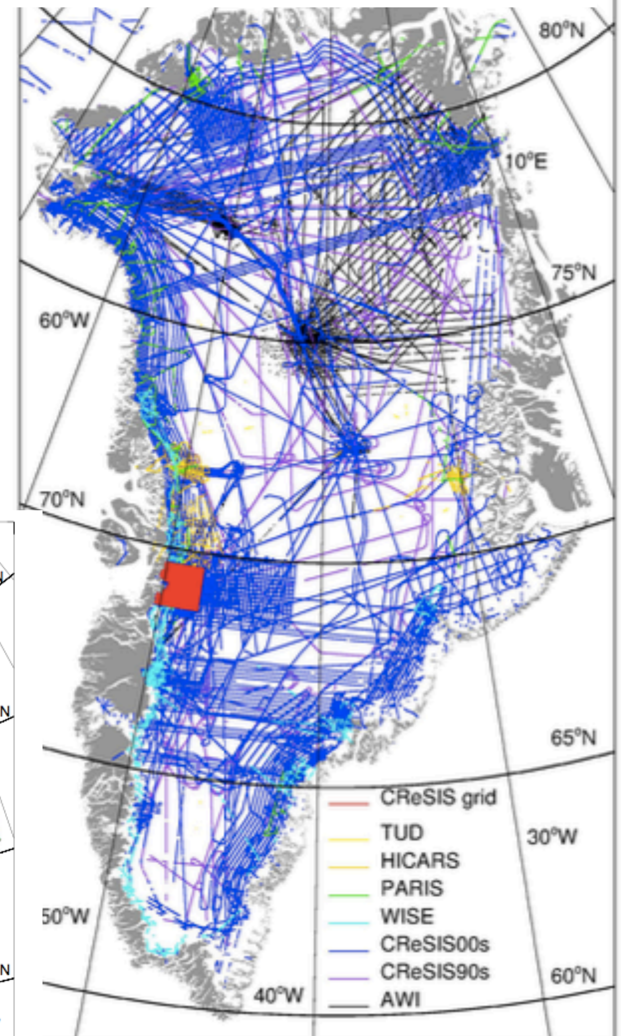


Fig. 7. RMS error in bed elevation due to the combined uncertainties in the IPR data and those due to interpolation.

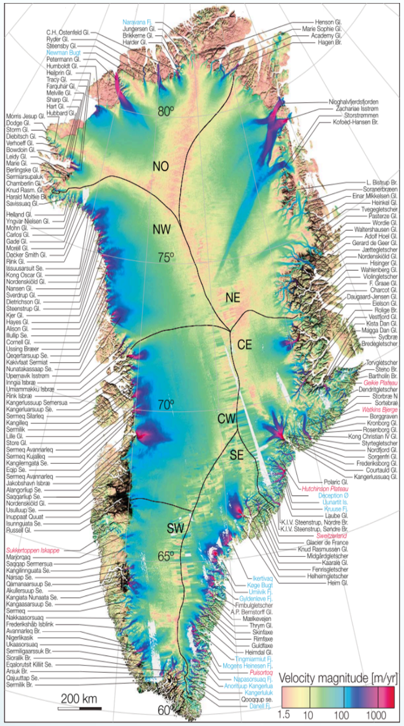
Airborne data lines
Source: [Bamber al.'13]

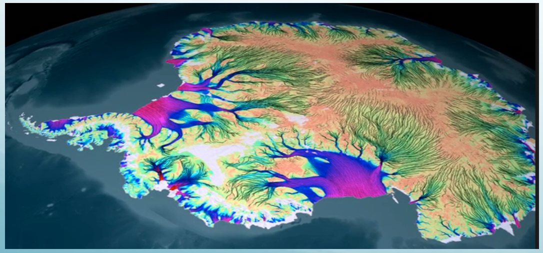


1. Data sources. CReSIS90 (purple) includes all data from 1993
99 as used in (Bamber et al., 2001b). CReSIS00 (blue) includes
ata derived from CReSIS instruments between 2000 and 2012.

Databases: InSAR, Altimetry & Airborne measurements

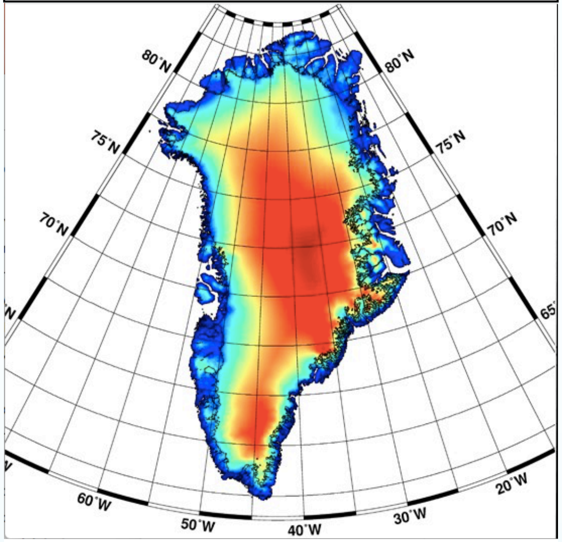
Ice velocities
derived from InSAR
[Rignot-Mouginot] et al.

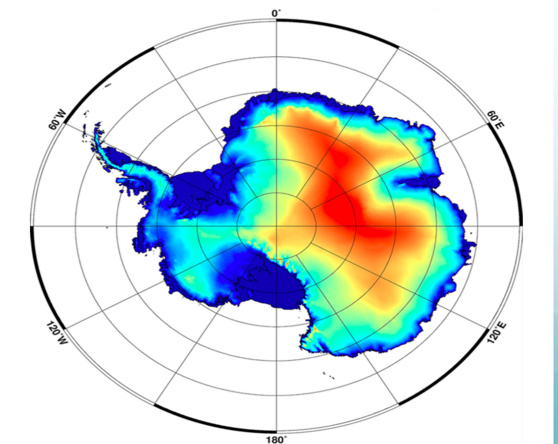




Top surface elevation (DEM) & Slopes
See eg [V. Helm et al.]'14 from Cryosat-2 (2012 data up to 88° N/S)

1-km grid DEM (error ~ a few m)
Slopes (~0.1° interior) at larger scale.





Airborne data

CReSIS et al.
[Bamber al.'13]

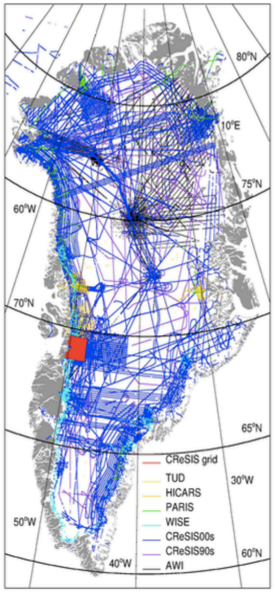
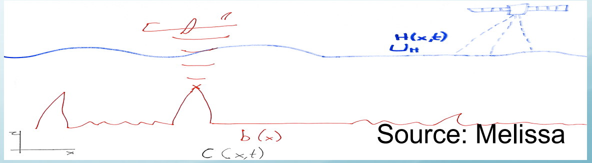


Fig. 1. Data sources. CReSIS90 (purple) includes all data from 1993 to 1999 as used in (Bamber et al., 2001b). CReSIS00 (blue) includes all data derived from CReSIS instruments between 2000 and 2012.



Bed topography inference: the current reference method [Morlighem et al]'14

Greenland bed topography maps by [Morlighem et al]'14

Streams: Depth-averaged mass equation
Interior sectors: [Bamber et al' 13]

Method : Combination of the depth-averaged mass equation (transport) (Rasmussen'88) + data cocktail (altimetry, InSAR, airborne) + Variational Data Assimilation (VDA).

➔ Topography in fast flow areas (~plug flows at ~100+ m/y) eg Greenland grid ~400m resolution.

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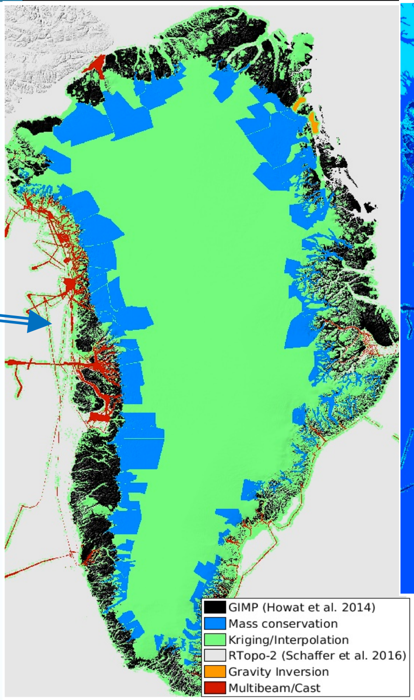
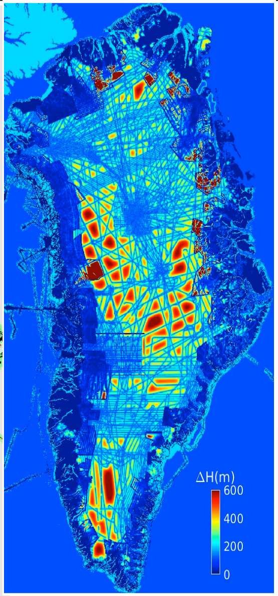
Pros: - Efficient since VDA + mass conservation.
- Extremely useful since in the fast flow areas.

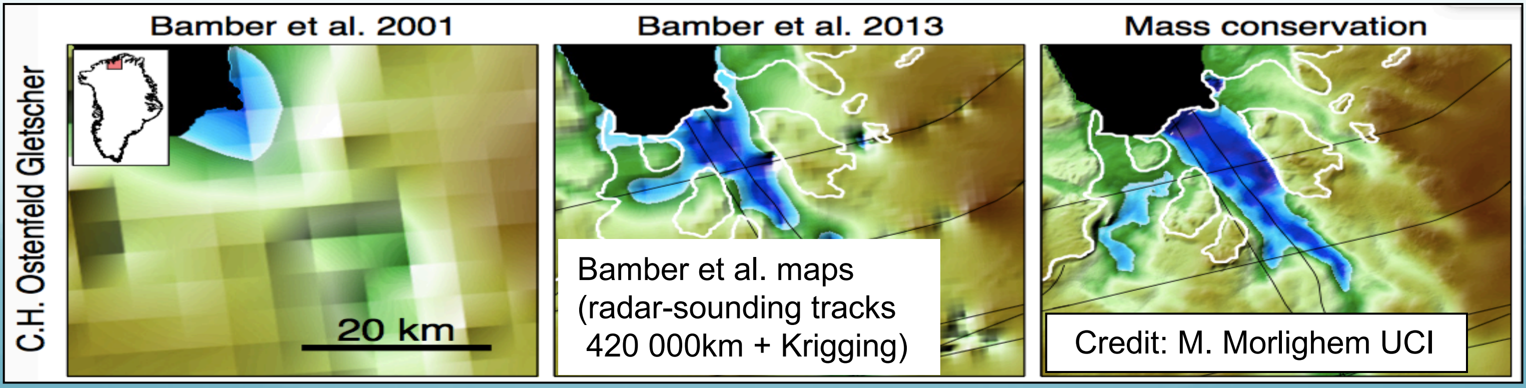
Cons:
- In the fast flow areas only since dynamic model ~ plug flow. No actual momentum conservation.
- Flight tracks at upstream are required.
- Error measurements are intrincally propagated since transport equation.





Interior: error up to 600m

➔ Accurate map in fast flow areas where quite dense flight tracks are available



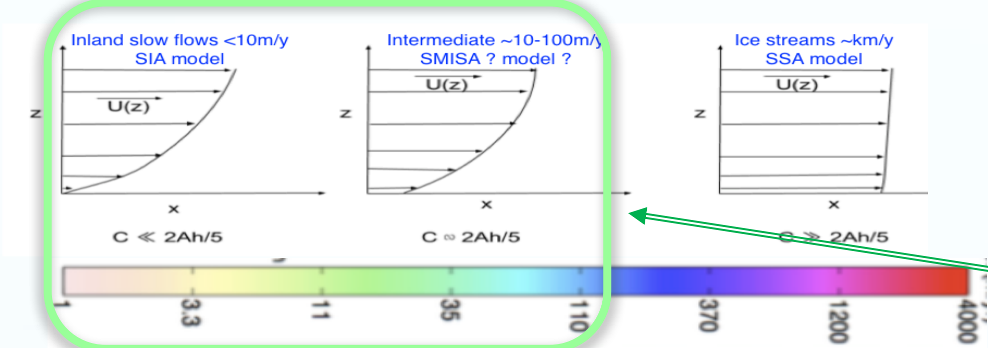
In view to infer the bed properties in poorly measured areas and/or in slow flow areas (< 50+ m/y)

Additional difficulties :

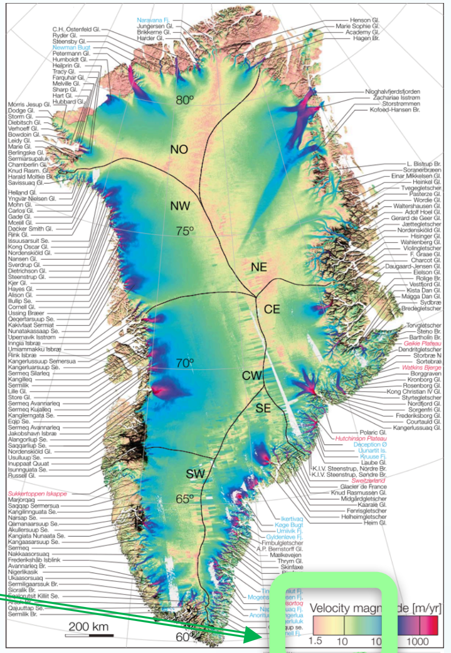
- The validity range of the **shallow ice flow models** is restricted

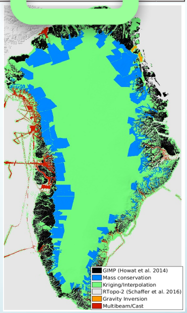
  The mass equation is not sufficient in sheared flow sectors.

The considered physical-based model

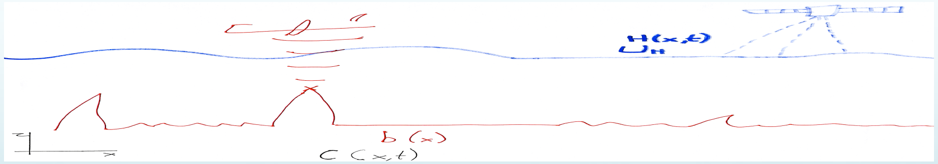


From fully sheared (grounded, ~m/y) to …. pure slip (fast streams ~km/y)





- **Separating the topography effects from the friction coefficient effects** measured from an unique signature: the surface observations.



Friction coefficient C = rough macroscopic model of the non-homogeneous material beneath the glaciers and/or subglacial hydrology.

Propagation of error measurements : better to damp than transporting

The proposed method to infer the bed properties in the interior sectors

From [Monnier-desBoscs]'17

Ingredients

- **A quite complete physics (xSIA, standard)**

Shallow Ice Approximation with weak/moderate slip at bottom

➔ Valid for slip ratio R_S ~ [0.3 -1.] < 50+ m/y

and at ~ 5 km scale.

- **The mathematical inverse method : analytico-numerical**

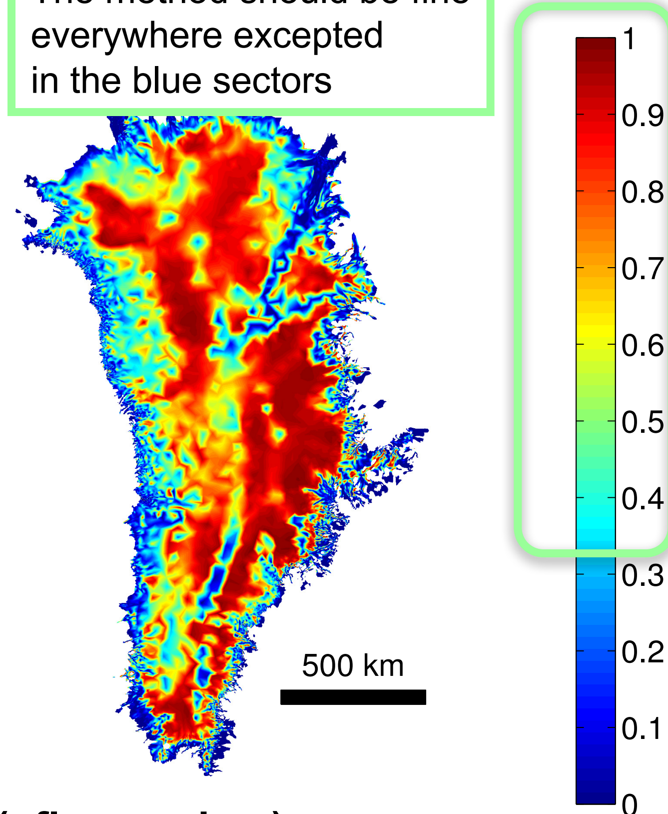
- Analytical calculations combined with the observed quantity

$\frac{\|U_H\|}{\|\nabla H\|^q}$: ratio InSAR / Altimetry

- VDA of Altimetry data H + Airborne data where available.

Fig. = Slip Ratio
From ISSM computations
H. Seroussi et al. JPL

The method should be fine everywhere excepted in the blue sectors



➔ Three depth estimations depending on the Slip Ratio (=flow regime)

$$h \approx \left[\frac{(q+1)}{2\bar{\rho}\underline{A}} \mathcal{Q}_H\right]^{1/(q+1)} \equiv h_{sr1}$$

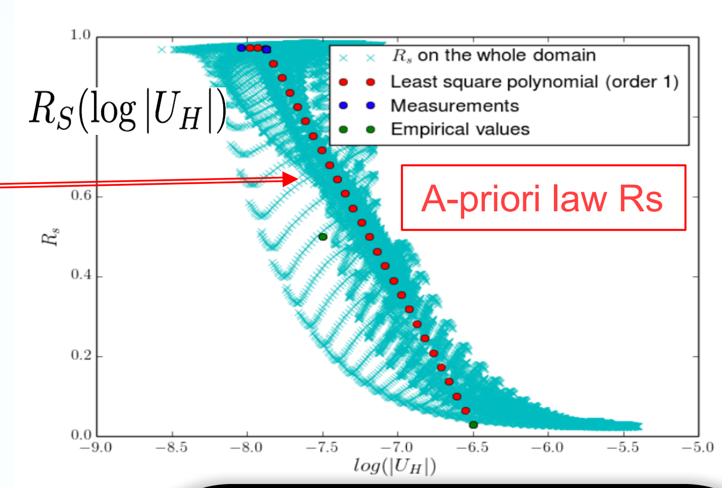
$$a_2 h^{q+2} - \frac{\mathcal{Q}_H}{\bar{\rho}} h + \eta = 0$$

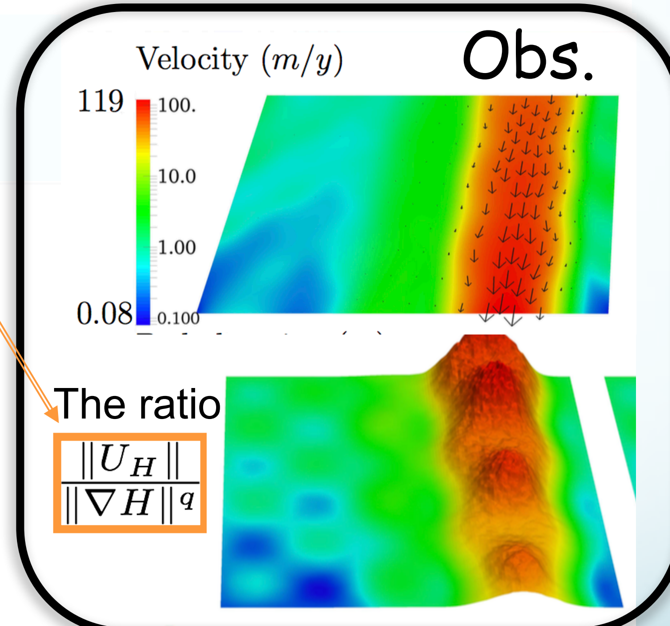
$$h = \left[1 - \frac{R_s}{(q+2)}\right]^{-1} \frac{\bar{\rho}}{\mathcal{Q}_H} \eta \equiv h_{sr2}$$

An academic multi-regime test case

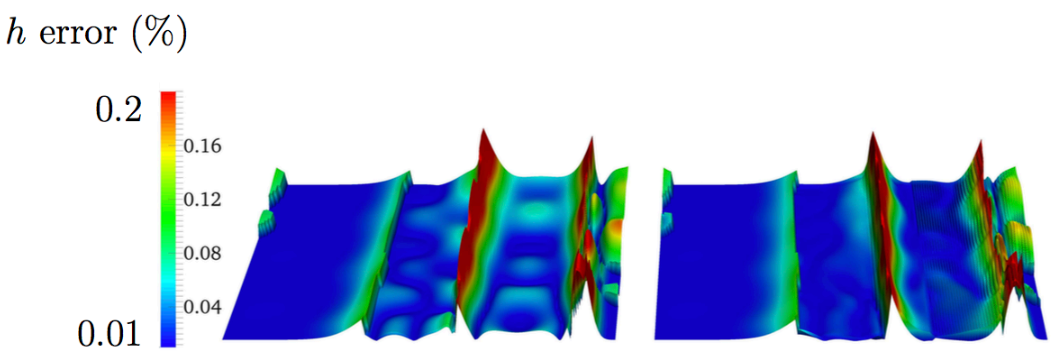
Step 1. A-priori slip ratio law $R_S(U_H)$
+ Analytical calculations
including the observed term $\frac{\|U_H\|}{\|\nabla H\|^q}$
➔ 1st good estimation of h (robust wrt Rs)

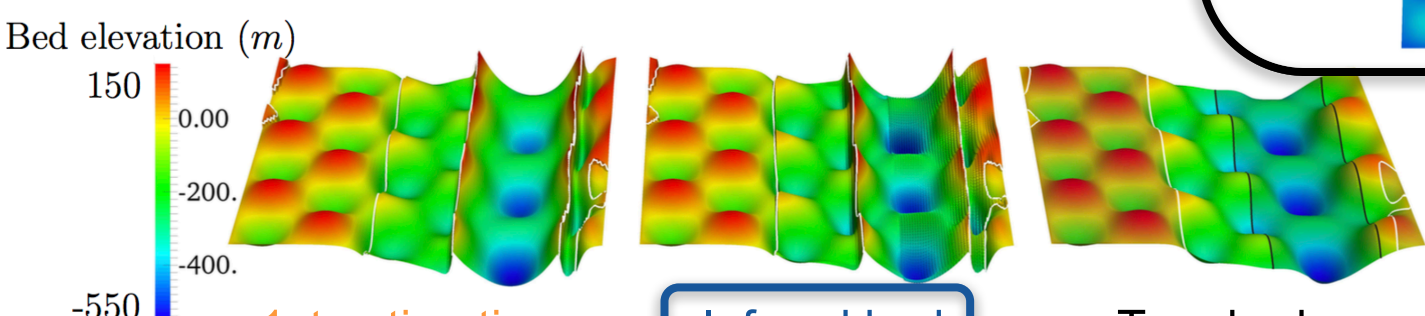
Step 2. VDA in H + polynom solved
➔ Infered bed





Errors ~ 5% & ~ 20% in stiff variations areas





1st estimation | Infered bed | True bed

The proposed inverse method

From [Monnier-desBoscs]'17

Pros

- Works where no other method does (it is dedicated to interior sectors < ~50+ m/y).
  Today: Kriging or gravity invertions with locally extremly large uncertainties.
- Can be performed even without any airborne measurement ! However more accurate if available !
- Works independently of the airborne measurement locations.
- Error measurements are damped and not propagated (elliptic equation and not transport-hyperbolic eqn)

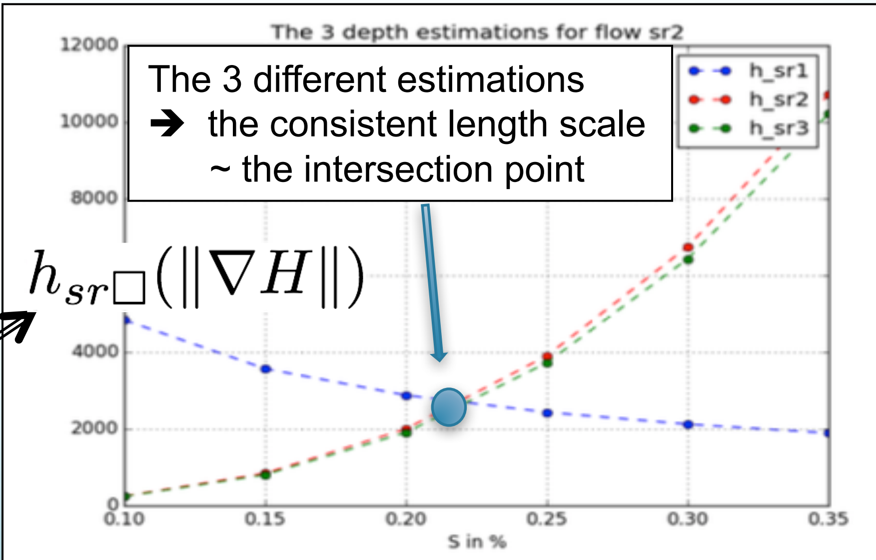
Cons

- Large scale resolution
  since based on a shallow model ➔ **~ 5-km grid**
- Highly sensitive to the slope scale value

  however there is a solution to this problem.

  Indeed from the 3 depth estimations…

Plot: The 3 depth estimations h vs slope values $\|\nabla H\|$
(Test with 20% noise on InSAR value of UH)



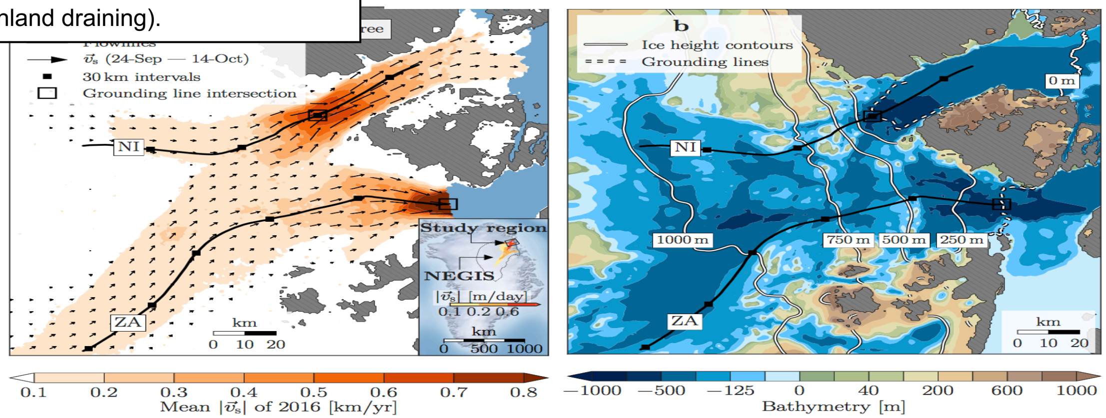
➔ The consistent length scale for the slope definition.

Study #2 : On the seasonal ice-streams modeling

Prospective study at its very beginning

Test case : Northeast Greenland Ice Stream
(16% Greenland draining).

Image source: Rathmann et al. GRL 2017



[Rathmann et al. 2017]'s study :

ESA Sentinel-1 SAR UH ~12 days repeat ➔ Analysis of the dynamic becomes possible.

Complex multi-physics dynamic : seasonal melting / sliding phenomena.
The multi-scale subglacial hydrology dynamics is not measured …
➔uncertain & complex sub-surface model (partially from observed dynamic surface lakes ~ km).

Our goal (prospective study !) :

Try to bypass this extremely difficult sub-surface modelling
by developing a multi-scale inverse method combined with HR surface data.

A good starting point is available :

Direct & inverse model: SSA equations with VDA of the complete data cocktail
including the High Frequency ESA Sentinel 1 data (swath SAR).
Ua computational software from H. Gudmundsson, British Antarctic Survey, UK.

British Antarctic Survey
NATURAL ENVIRONMENT RESEARCH COUNCIL

Physical-based inverse methods
to infer ice-sheets & ice-streams bed properties

* Forthcoming actions *

Study #1: Can the interior bed topographies be infered from surface data ? Let's try it !...

- ✓ Inverse method assessed into details for academic test cases (IMT).
- ❑ **Invertion from the complete databases planned in automn '17**

  with M. Morlighem (UCIrvine), ISSM computational software (UCI-JPL).

Study #2: On the seasonal ice-streams: multi-scale inverse approach. Propective study.

- ❑ Seasonal dynamics: need High Frequency & High Resolution data.

  eg Sentinel-1 SAR : the crucial data sets in the very recent & current studies.

- ✓ Up to now, *in a purely computational point of view,* our new inverse method is promising …

  (Preliminary study at IMT 2016-17).

- ❑ **How is it relevant to model the seasonal ice stream dynamics ?**

  ➔ One of our small research program goal CNES Tosca project.

  Postdoc begins in october'17

  In collaboration with H. Gudmundsson (BAS, UK)
  & Colleagues from IMT (optimisation).

Some physical-based inverse methods

to infer ice-sheets & ice-streams bed properties

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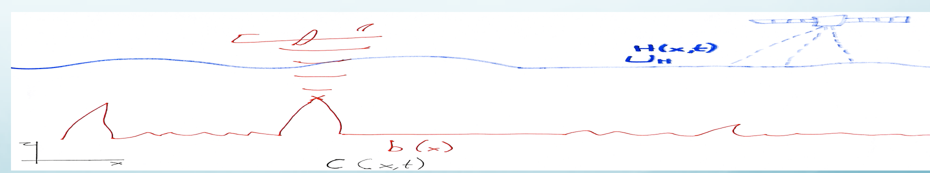
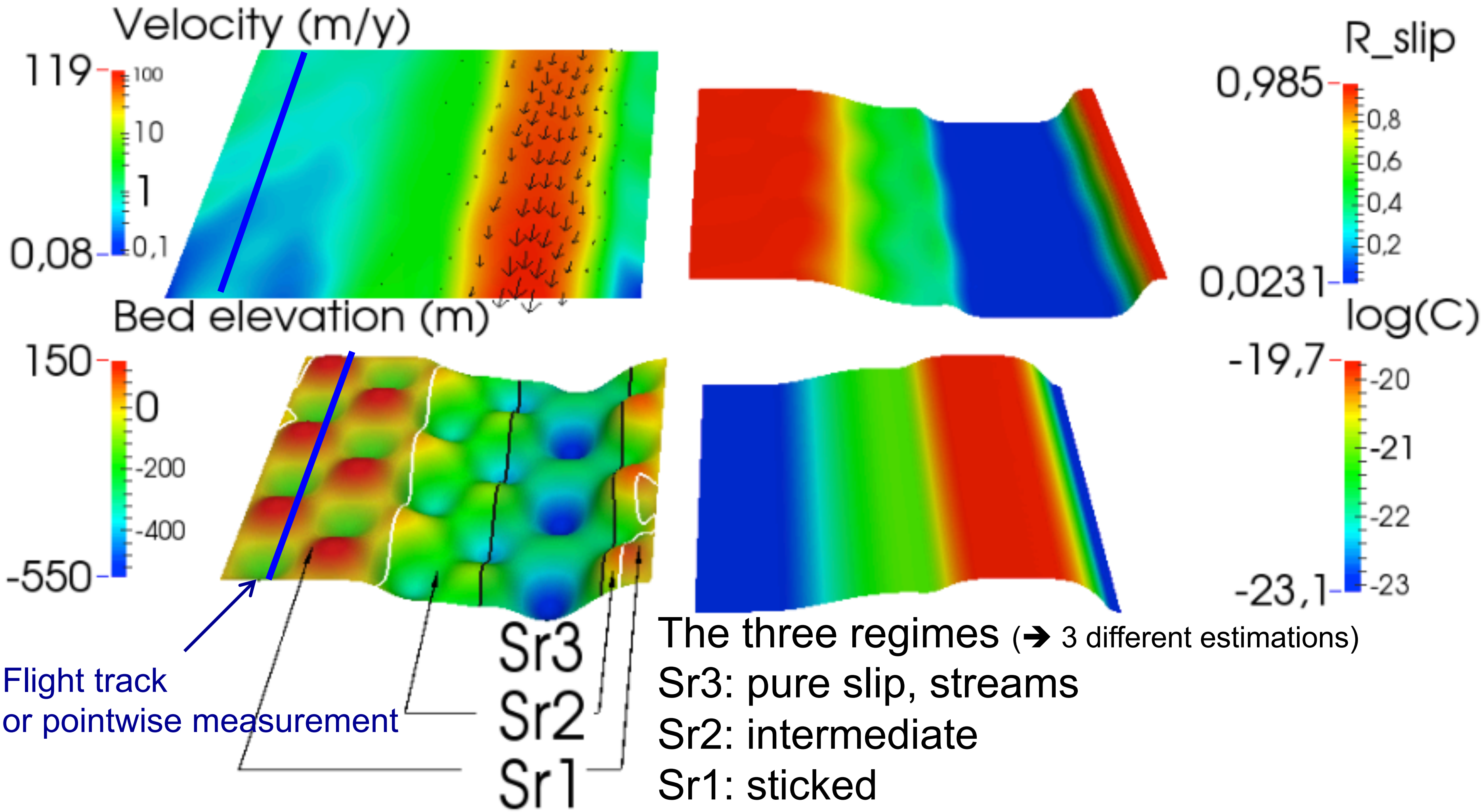
Thank you for your attention

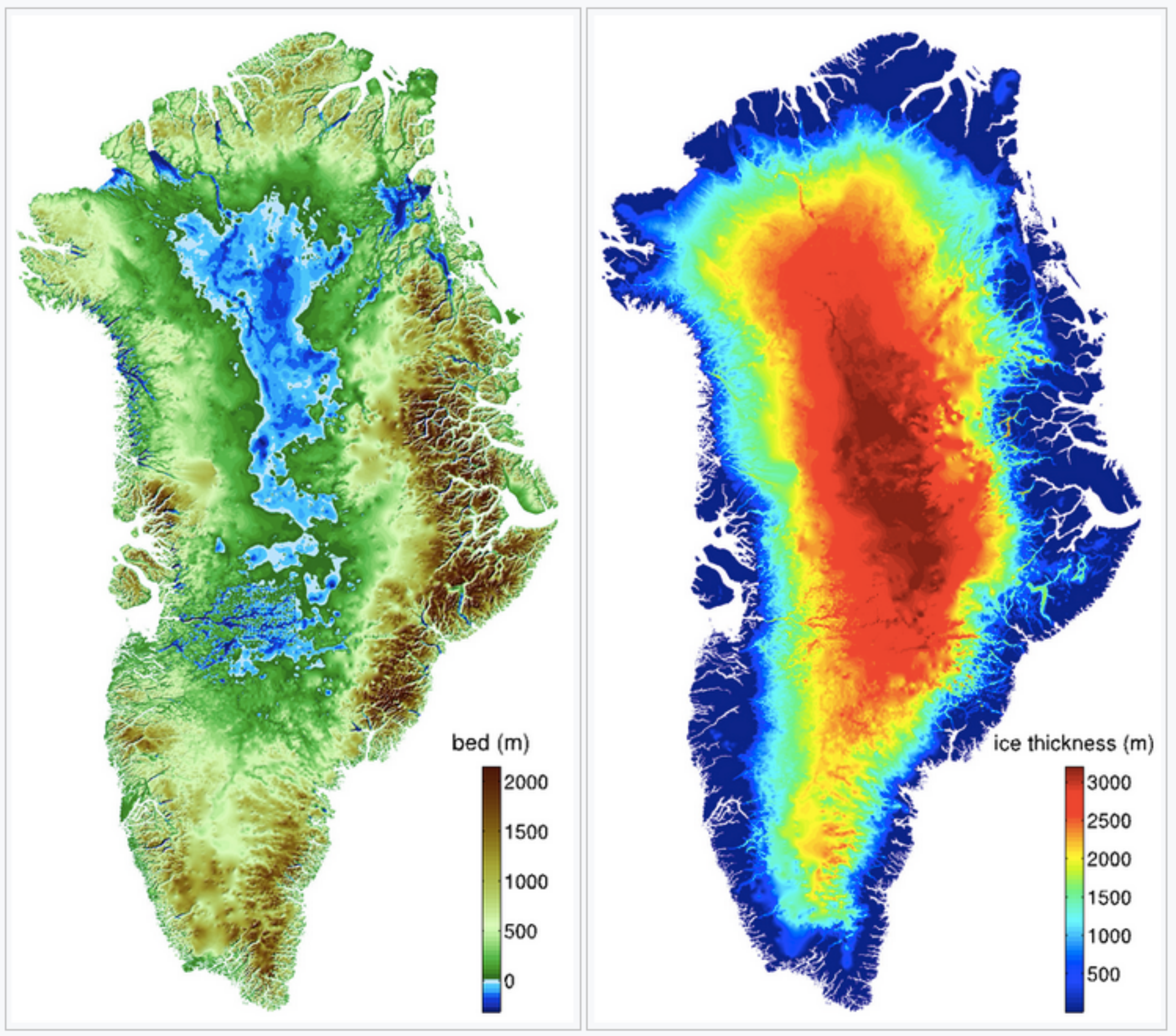


An academic multi-regime test case

- Academic but complex since fully multi-regime (+ real-like noise amplitudes).
- «Airborne» measurements ($\mathcal{Q}_H$: 20% noise) : lateral boundary (or pointwise !)
  ➔ case not resolvable by the existing methods.



The three regimes (➔ 3 different estimations)
Sr3: pure slip, streams
Sr2: intermediate
Sr1: sticked



Greenland bed topography maps
by [Morlighem et al'14]
Streams: Depth-averaged mass equation
Interior sectors: [Bamber et al' 13]

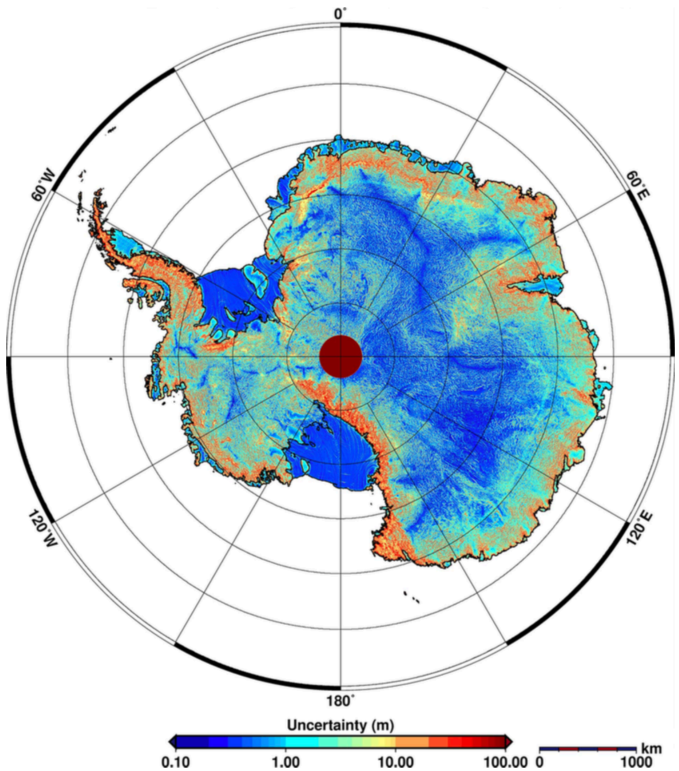


Figure A1. Uncertainty map of the new Antarctica DEM calculated using a multiple regression approach based on DEM–ICESat differences.

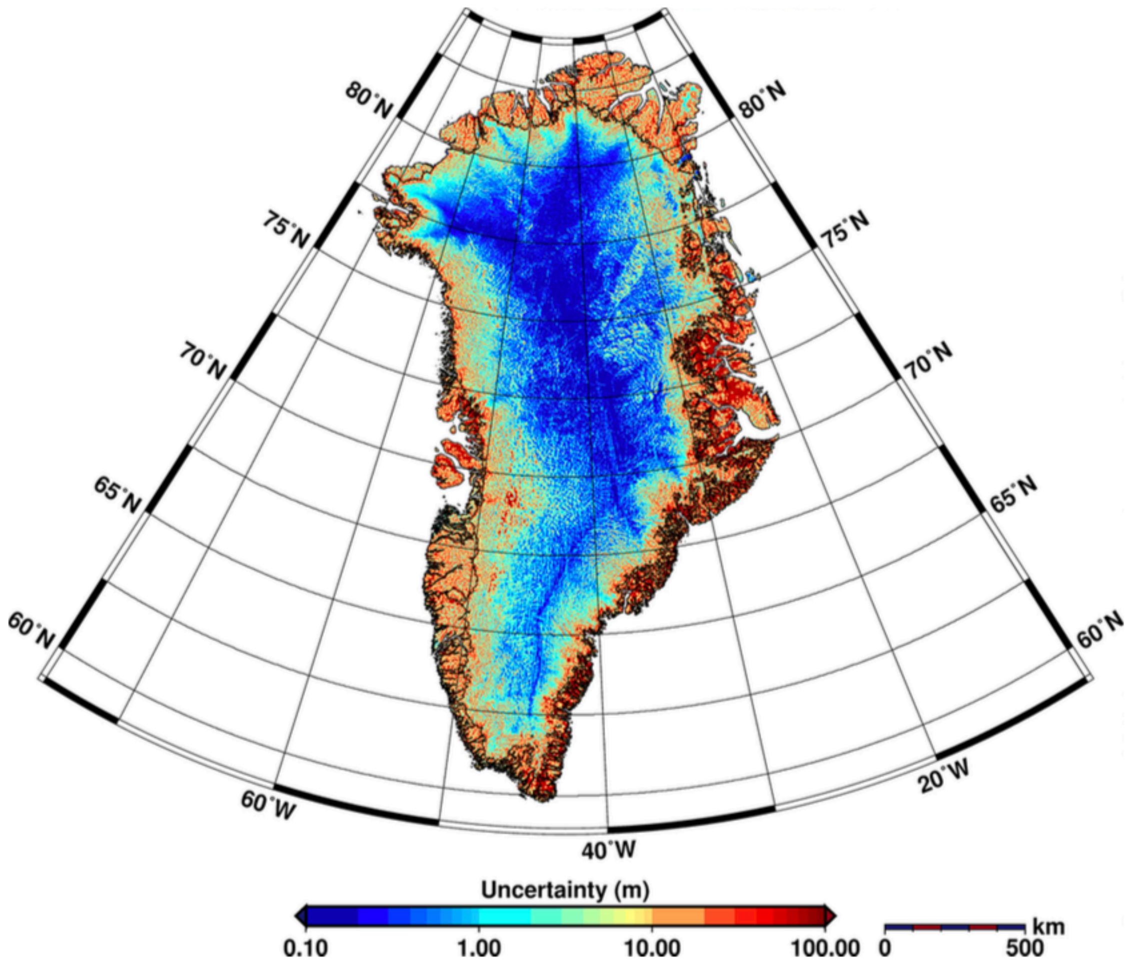


Figure A2. Uncertainty map of the new Greenland DEM calculated using a multiple regression approach based on DEM–ICESat differences.

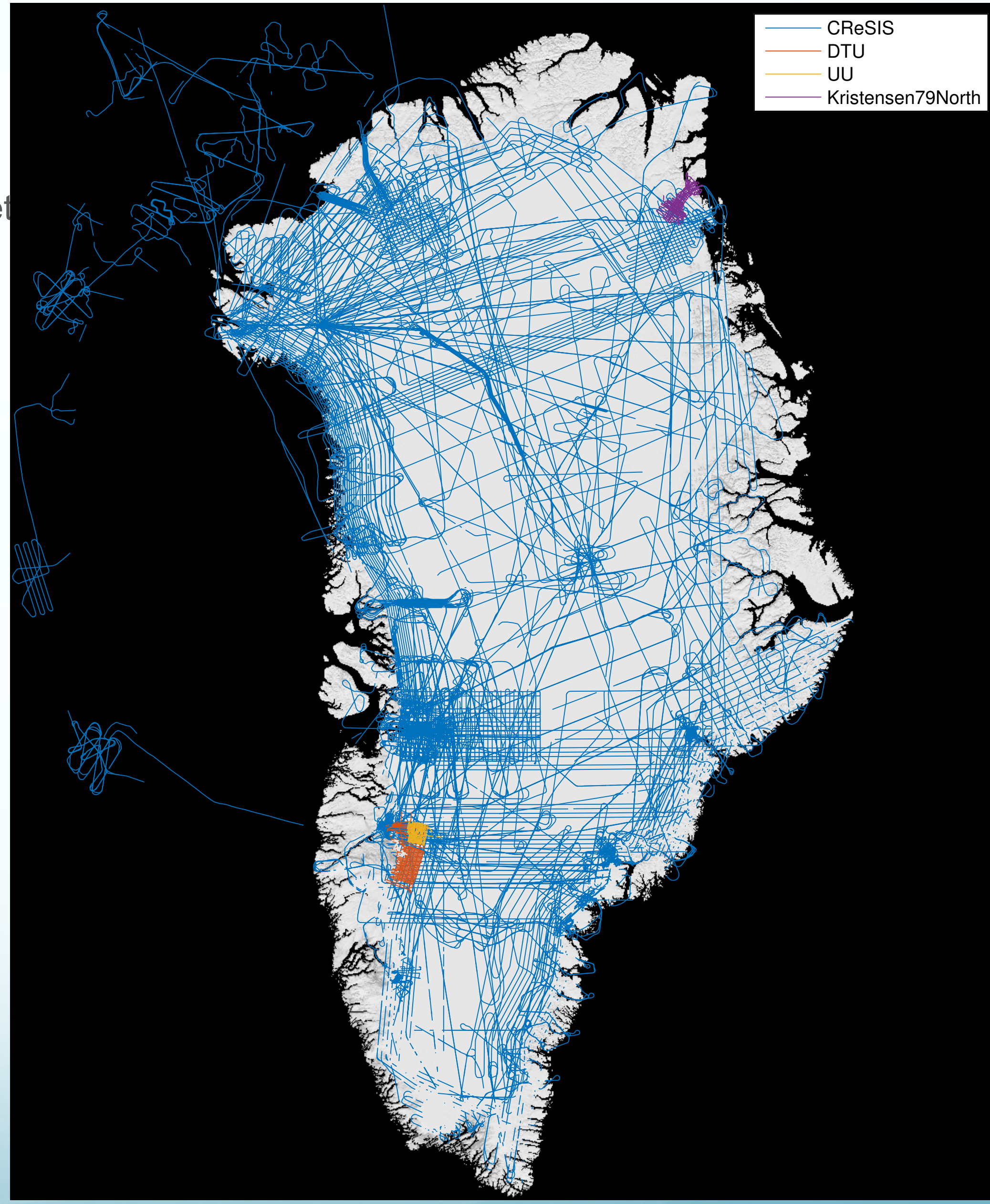
CReSIS = Center for Remote Sensing

of the Ice Sheet (Kansas University) (NASA, SSF, et

DTU = Technical University of Denmark

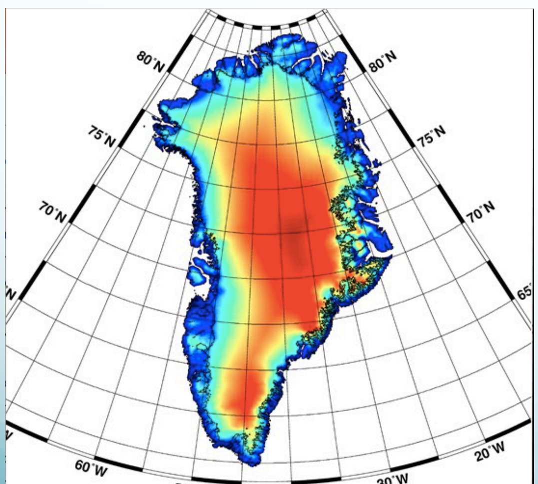
UU = Utrecht University

Map: M. Morlighem pers. Comm

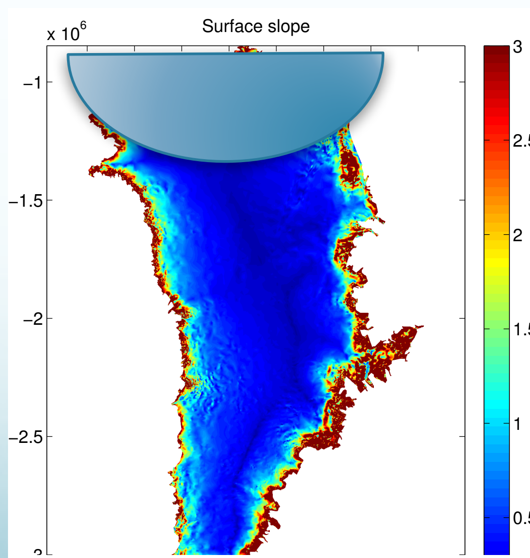


Satellite altimetry

	Sentinel-3	ICESat 2	SWOT
Launch	2016	2017	2021
Footprint	300 m Nadir over SeaIce	10 m footprint (4 lasers)	120 km swath
Repeat	27 days with (up to) 10 repeats at high latitude	30/90 days	22 days with up to 12 repeats at high latitude (78o max)
Accuracy	? (SSH: 3-5cm)	15 cm	?
Limitations	Snow/ice penetration ?	3 km space tracks Cloud coverage	Snow/ice penetration ?



Top surface elevation
from ESA Cryosat-2
[V. Helm,Cryos.'14]



Surface slopes from…
Credit: Seroussi-Larour, JPL

High frequency visits
➔ precious data
for rapidly varying coastal flows
e.g. Southern Greenland with seasonal dynamics

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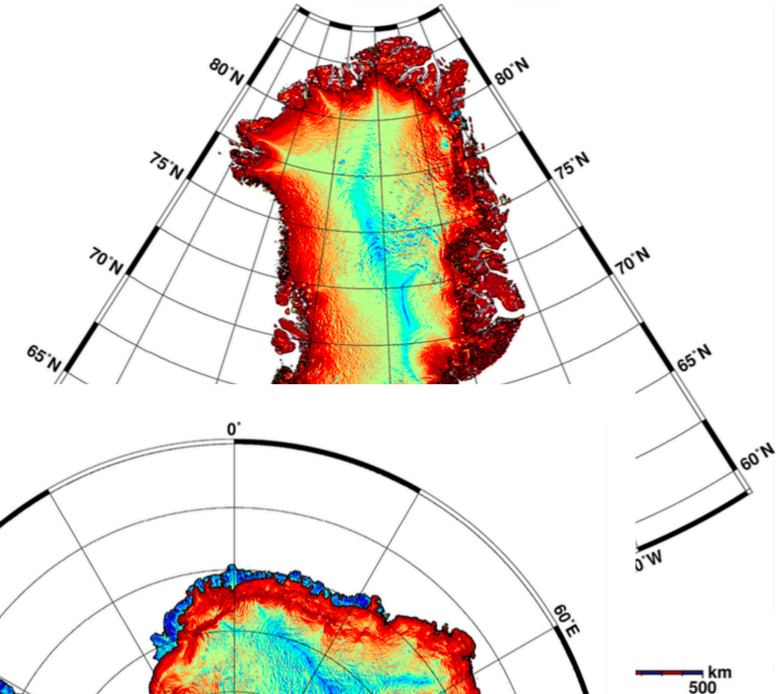
SWOT
LR inland
HR coastal flows (?)

From Helm TC 2014
CryoSar-2



ation model

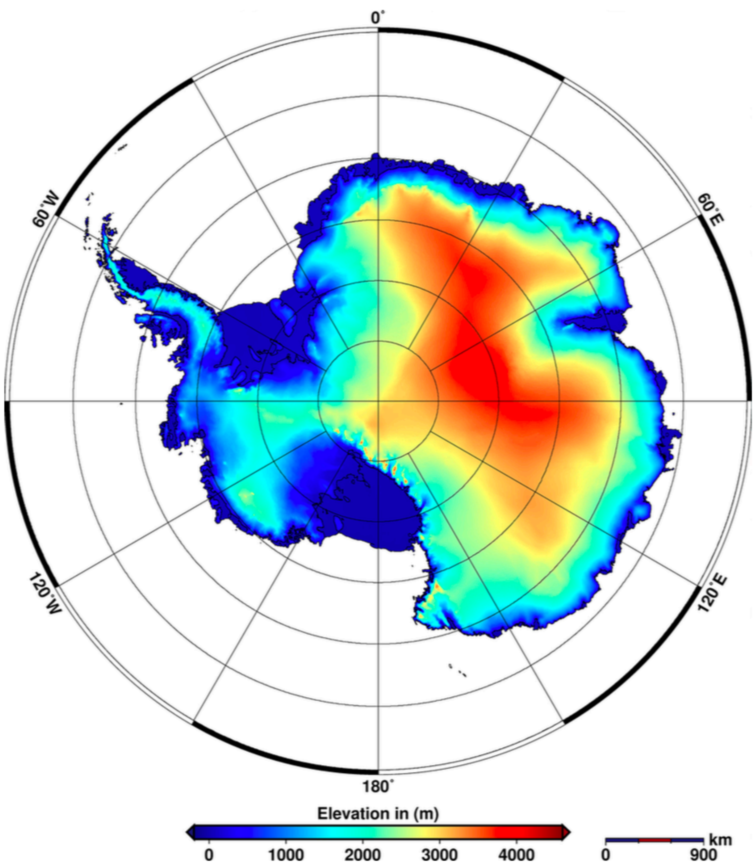


Figure 5. New elevation model of Antarctica derived from CryoSat-2.

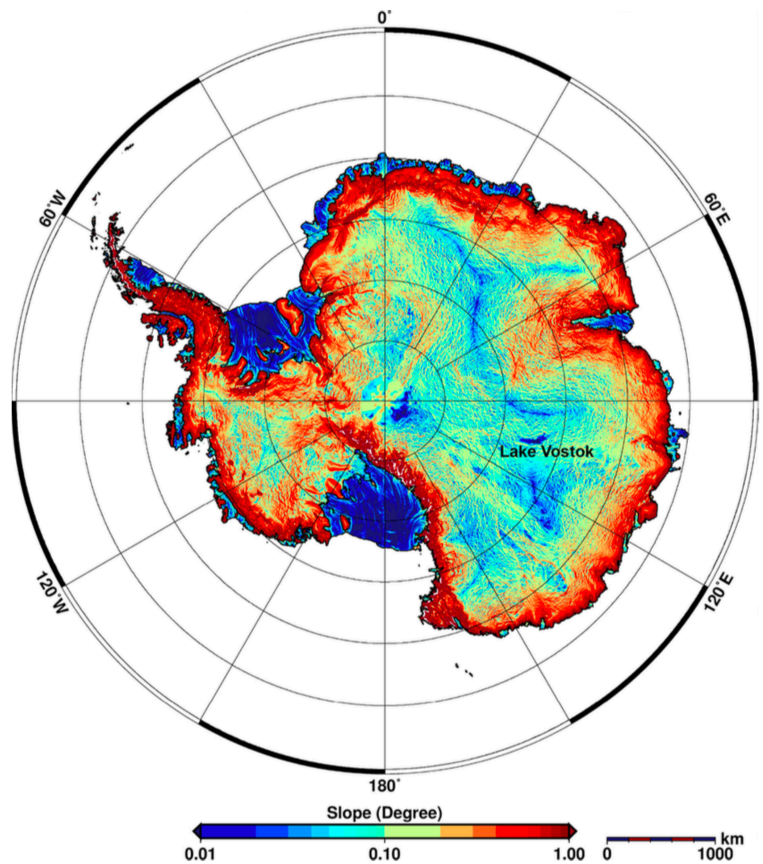


Figure 6. Surface slopes, estimated from the new elevation model of Antarctica.

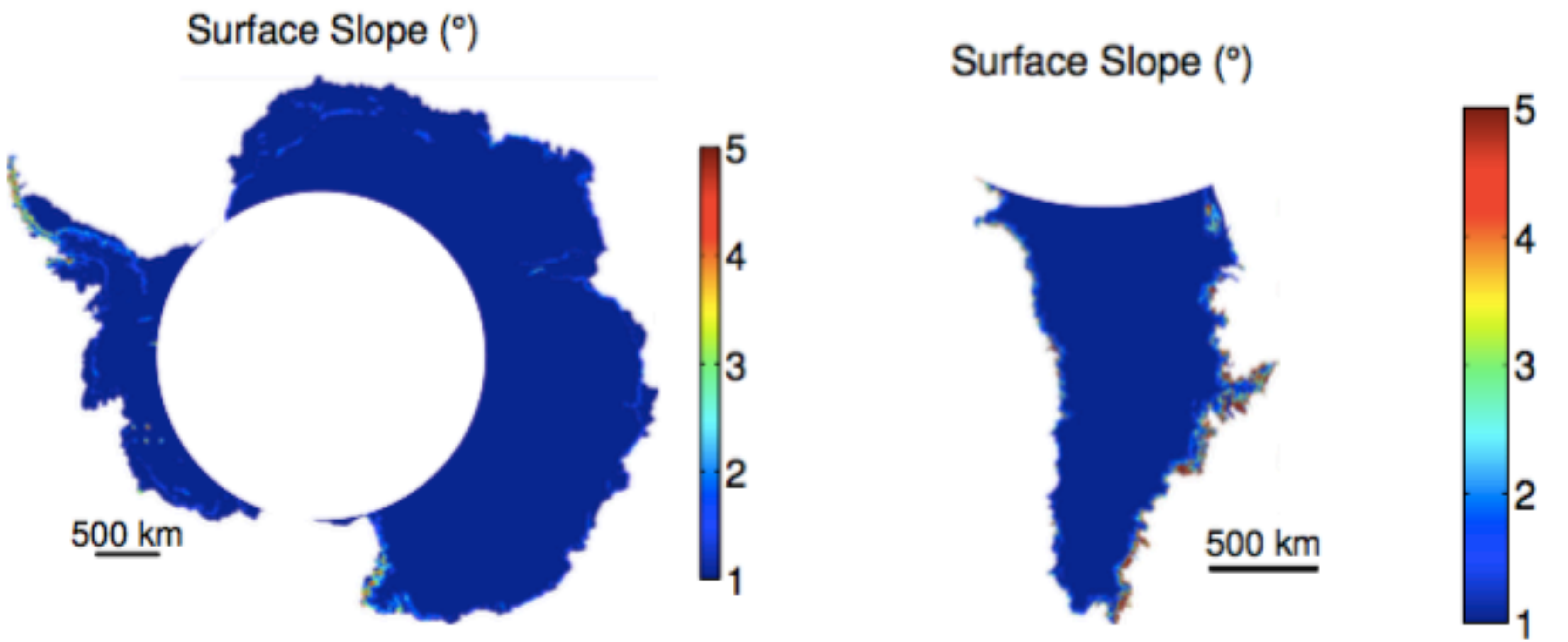


Figure 1. Estimated coverage of SWOT over Antarctica (left) and Greenland (right): surface slopes (in °) and radar coverage. The white zones correspond to areas not covered by the radar

Image: courtesy of H. Seroussi, JPL

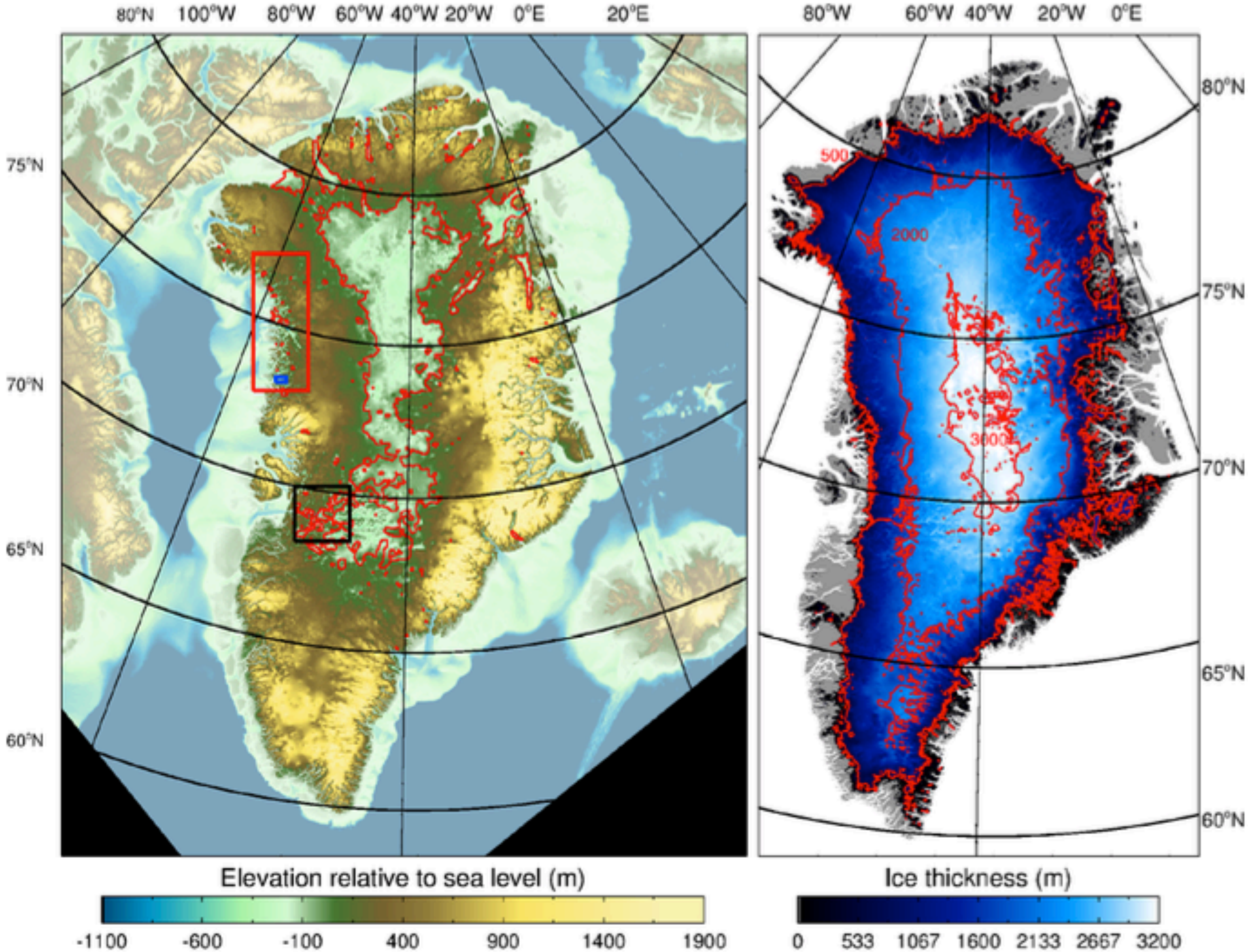


Fig. 3. **(a)** Bed elevation and bathymetry. Bathymetry is plotted on an opaque scale. Black and red boxes show the location of the Jakobshavn and north-west coastal regions shown in Fig. 4a–f, respectively. The blue box shows the location of the region shown in Fig. 2. The red contour is at 0 m above sea level. **(b)** Ice thickness as determined from the difference between surface and bed DEMs, with contours at 500, 2000 and 3000 m.

From [V. Helm,Cryos.'14]
Data Cryosat-2 ESA 2012