Surface Water and Ocean Topography (SWOT) Mission

SWOT 2nd Science Team Meeting

June 26-28, 2017

Revised error budget for wave effects

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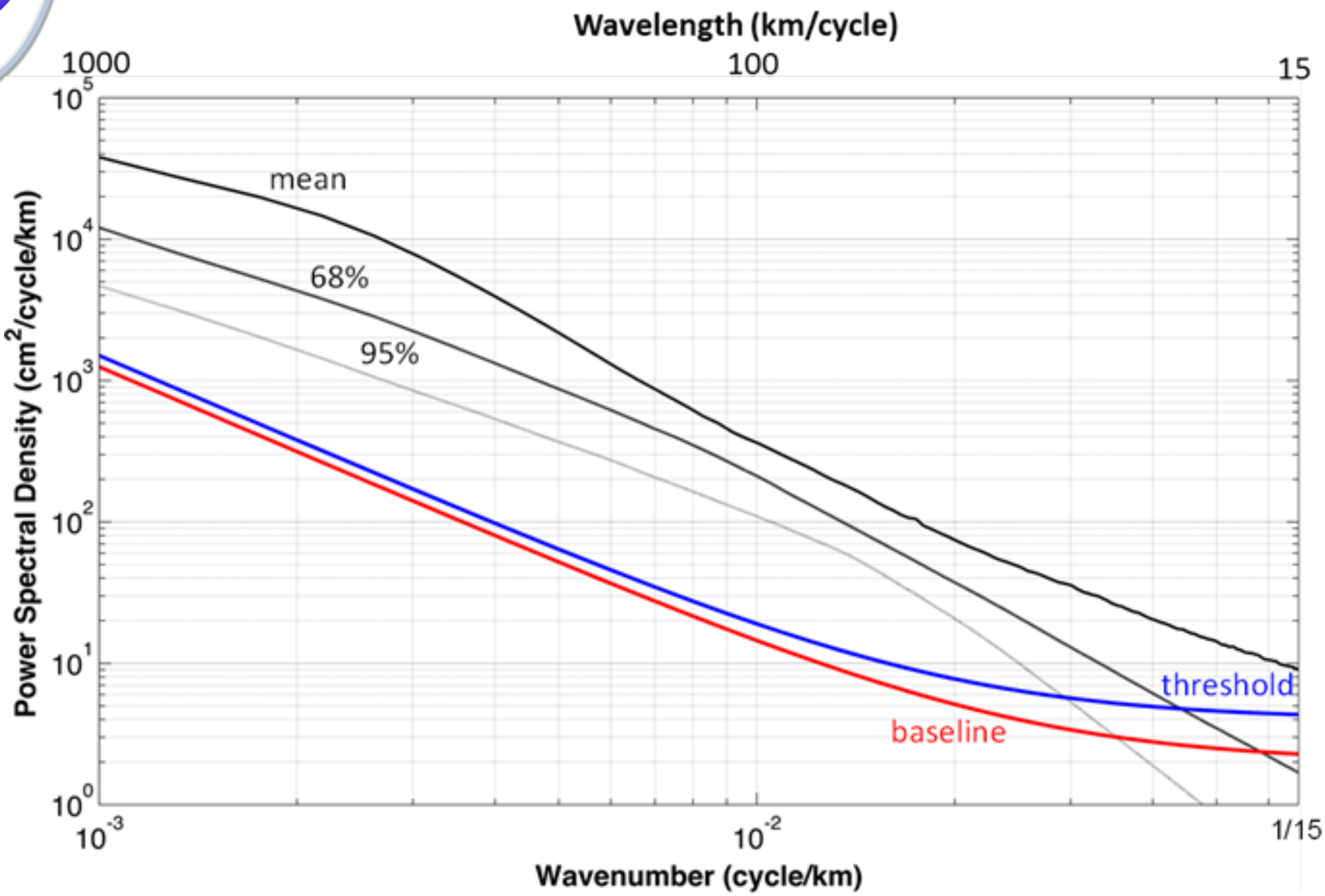
Introduction

- The SWOT error budget is documented in the SWOT Mission Error Budget Document, JPL D-79084.
  - Describes the measurement, and documents the entire derivation and flow-down of the error budget.
  - Document underwent significant expansion, and Revision A was recently released and made available on http://swot.jpl.nasa.gov
- In this presentation, we discuss the addition of motion errors to the overall ocean error budget.

Ocean SSH Requirement <1,000 km



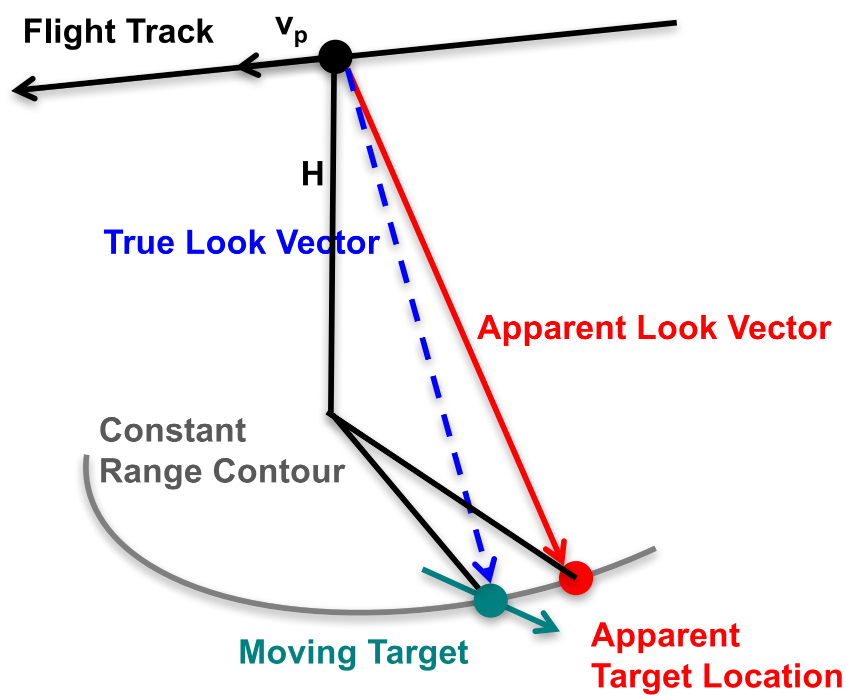
The fundamental topographic measurement is provided by KaRIn, as a swath measurement

Ocean Error Budget Drivers

- Ocean SSH requirements apply over short time scales (1,000 km or ~2.6 min) and are described as a PSD.
  - Drives overall stability of the flight system over these time scales
  - Drives a minimum signal-to-noise ratio in the KaRIn measurement
  - Also need to consider other error contributors:
    - POD: radial height errors
    - Media effects: causing propagation delays of the EM signal: wet tropo, dry tropo, ionosphere
    - Sea-State Bias: different scattering from troughs and peaks of the waves bias the measured height
    - Wave effects (volumetric decorrelations, surf-board effect, motion errors)

Velocity Effects on Interferometric Height (I)

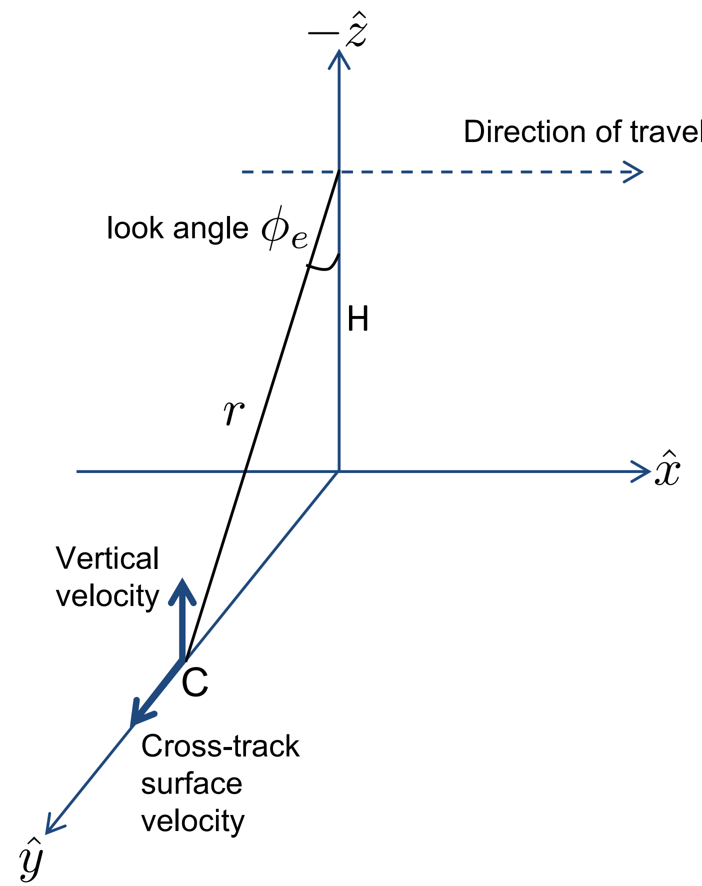
- Target motion causes errors in the interferometric height estimate
  - This is a well known effect: moving targets appear shifted in SAR images
  - Without knowledge of (and correction for) the target motion, the difference in interferometric phase between targets at the true and apparent target positions becomes an error in the interferometric measurement.
    - Second-order effect but SWOT's required accuracies are challenging enough that higher-order effects cannot be ignored.
  - In addition, along-track shifts from spatially variant target motion (e.g., waves) cause height errors through spectral distortion ("wave bunching")



- Motion errors are a fairly recent development
  - Emerged as a result of recent AirSWOT observations and data analysis
  - For SWOT, the pointing control error is sufficiently small to ensure that velocity errors are small, and the complex averaging over 7.5 km cross-track mitigates the impact of wave bunching such that the motion errors can be absorbed.

Velocity Effects on Interferometric Height (II)

- Two separate effects: cross-track surface velocities and vertical velocities.
  - Both velocities couple as a measured radial velocity (the projection onto the look vector), i.e. a Doppler shift.
  - However, given the near-nadir geometry, their projections are very different:
    - ♦ Surface velocities couple with a sin(look angle) = C/*r*, which is small (0 at nadir).
    - ♦ Vertical velocities (due to the circular, or orbital motion of the waves), couple with a cos(look angle), which is not small.
  - The Doppler shift is proportional to the ratio of the surface radial or cross-track velocity v to that the platform velocity v_p, v/v_p, for $v_p >> v$



H ~ 900 km
Look angle ~ [0.4 to 4] deg
for C = [10, 60] km

Velocity Effects on Interferometric Height (III)

- With perfect attitude and a perfect yaw-steered, undistorted baseline, the impact of surface motions for SWOT is negligible (<0.1 mm)
- In the presence of attitude control errors, the dominant error is given by a pitch, introducing, for cross-track velocities, a swath-average height error:

$$\delta h = \bar{C} \frac{v_s}{v_p} \alpha_{pitch}$$

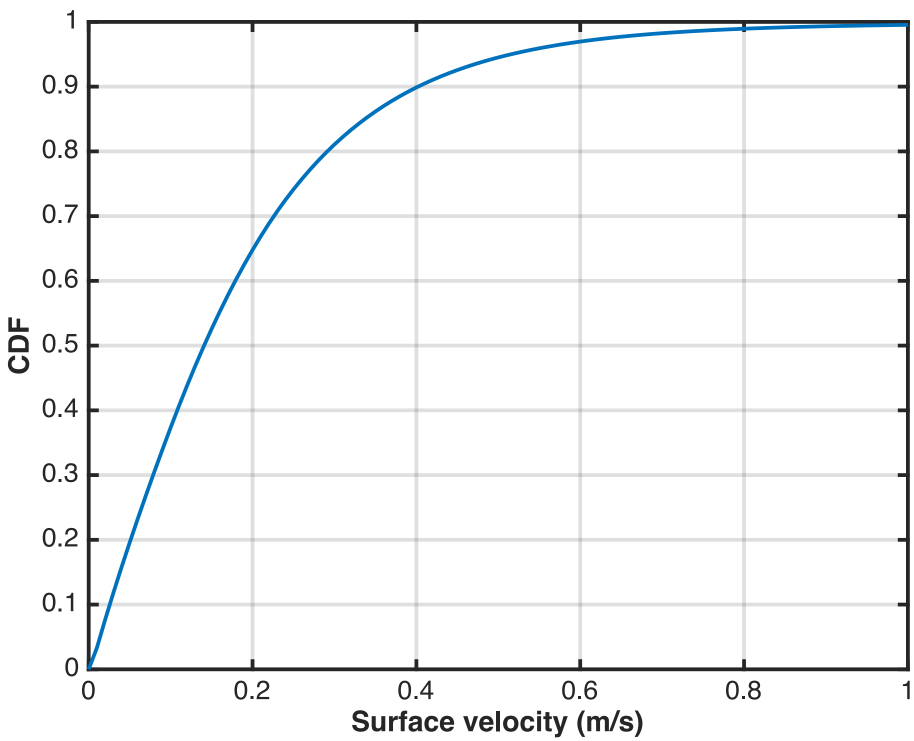
- Similarly, for a radial velocity, the pitch error dominates introducing a height error:

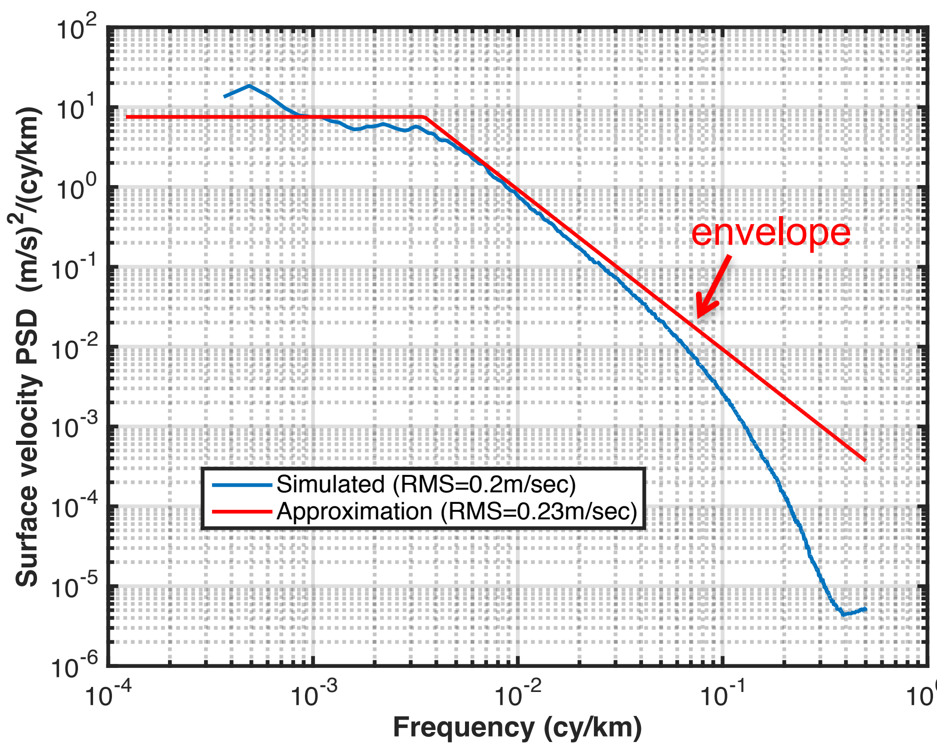
$$\delta h = H \frac{v_r}{v_p} \alpha_{pitch}$$

- Given the difference in H to $\bar{C}$ is ~900 km to 38 km, and considering ocean surface velocities of ~0.2 m/s and ocean radial velocity is ~0.1 m/s, ***radial velocities dominate the height error by a factor of ~10***.

Velocity Effects on Interferometric Height (IV)

- Since for the ocean the SSH error is expressed in the form of a PSD, we derive upper bound for the PSD of the radial and cross-track velocity errors.
- For <u>cross-track velocities</u>, we used the latest version of the ECCO-2 global model to derive a PSD of the ocean surface velocity, and extract an spectral upper bound.
  - Used *u* and *v* components of the surface velocity from the model, and computed the projection in cross-track along the orbit according to the nominal heading





Mean Brightness Modulated Radial Velocity

- The ocean radar brightness given by Geometric Optics:

incidence angle

reflection coefficient

$$\sigma_0(x) = \frac{R^2}{s^2(x)} \exp\left[-\frac{(\theta - \zeta_x)^2}{s^2(x)} \right]$$

large scale slope

mean squared slope (mss)

is well known to present several brightness modulations:

- ***Hydrodynamic modulation***: dependent on the mean squared slope (mss) of the small waves:

$$s^2(x) = \bar{s}_0^2 \left[1 + m_H \cos\left(\Theta + \phi_m\right) \right] \qquad \phi_m \in \ [200, 240] \ \text{deg}$$

mean slope variance

magnitude of the hydrodynamic transfer function

phase of the modulation; can be in phase with height or velocity depending on modulation phase

- ***Tilt modulation***: dependent on the local incidence angle and modulated by large wave slopes

tilt modulation function; face of the wave facing the radar significantly brighter than the one facing away from the radar.

$$\exp\left[-\frac{(\theta - \zeta_x)^2}{s^2(x)} \right] \approx \exp\left[-\frac{(ak)^2}{2s^2} \right] \exp\left[-\frac{\theta^2}{s^2} \right] \overbrace{\left(1 - 2\frac{ak}{s}\frac{\theta}{s}\sin\left(\Theta\right) + \frac{1}{2}\left(\frac{ak}{s}\right)^2 \cos\left(2\Theta\right) \right)}$$

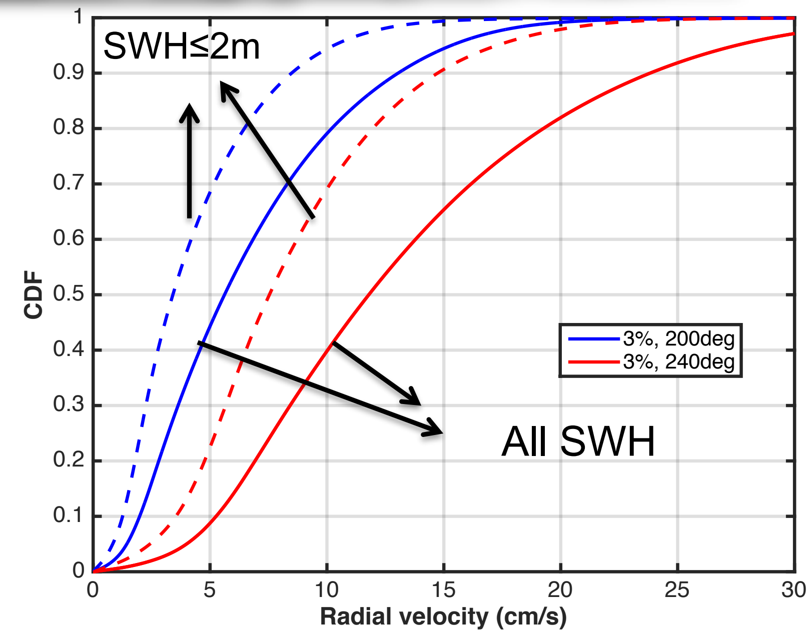
90° out of phase with elevation and in phase with the velocity

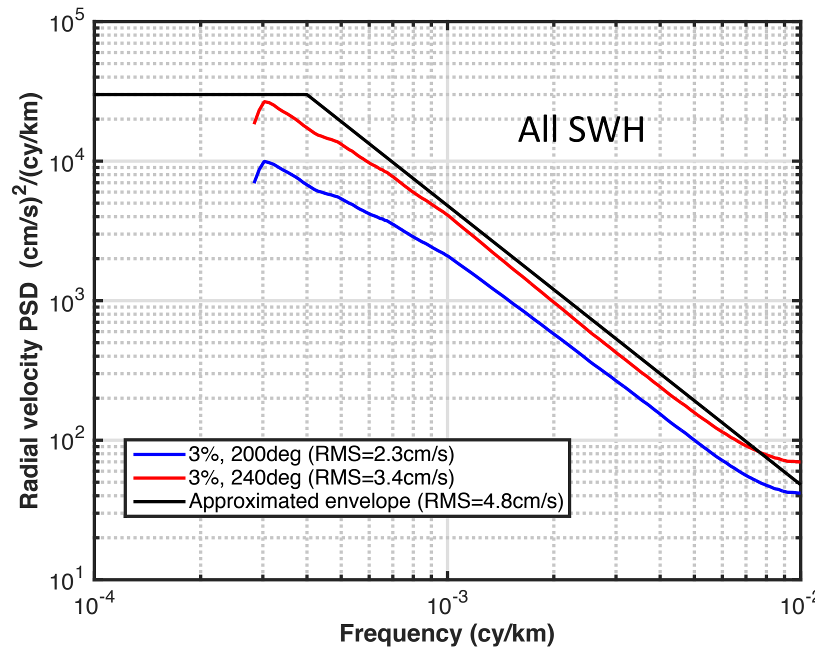
Mean Brightness Modulated Radial Velocity

- We consider the magnitude of the hydrodynamic transfer function to be consistent with a 3% EM bias, and the phase term consistent with observations in the literature (180-240 deg).
  - Likely conservative: initial AirSWOT observations indicate that at SWOT angles, the brightness modulation is dominated by the tilt modulation, with hydrodynamic modulation playing a small role, reducing the impact of the velocity errors.
- Used a year of Global WaveWatch-3 realizations to obtain PSD of radial velocities.
  - From WW-3, we extract the primary wave mean wavelength and SWH, from which the u (horizontal) and ω (vertical orbital) components are computed assuming monochromatic wave, and used the theory to obtain the brightness-modulated heights.

$$\langle v_r \rangle_{\sigma_0} = \int_0^{2\pi} M \, v_r \, d\Theta$$

  - 68 % for SWH ≤ 2 m is ~10 cm/s, and 16 cm/s for all SWHs.





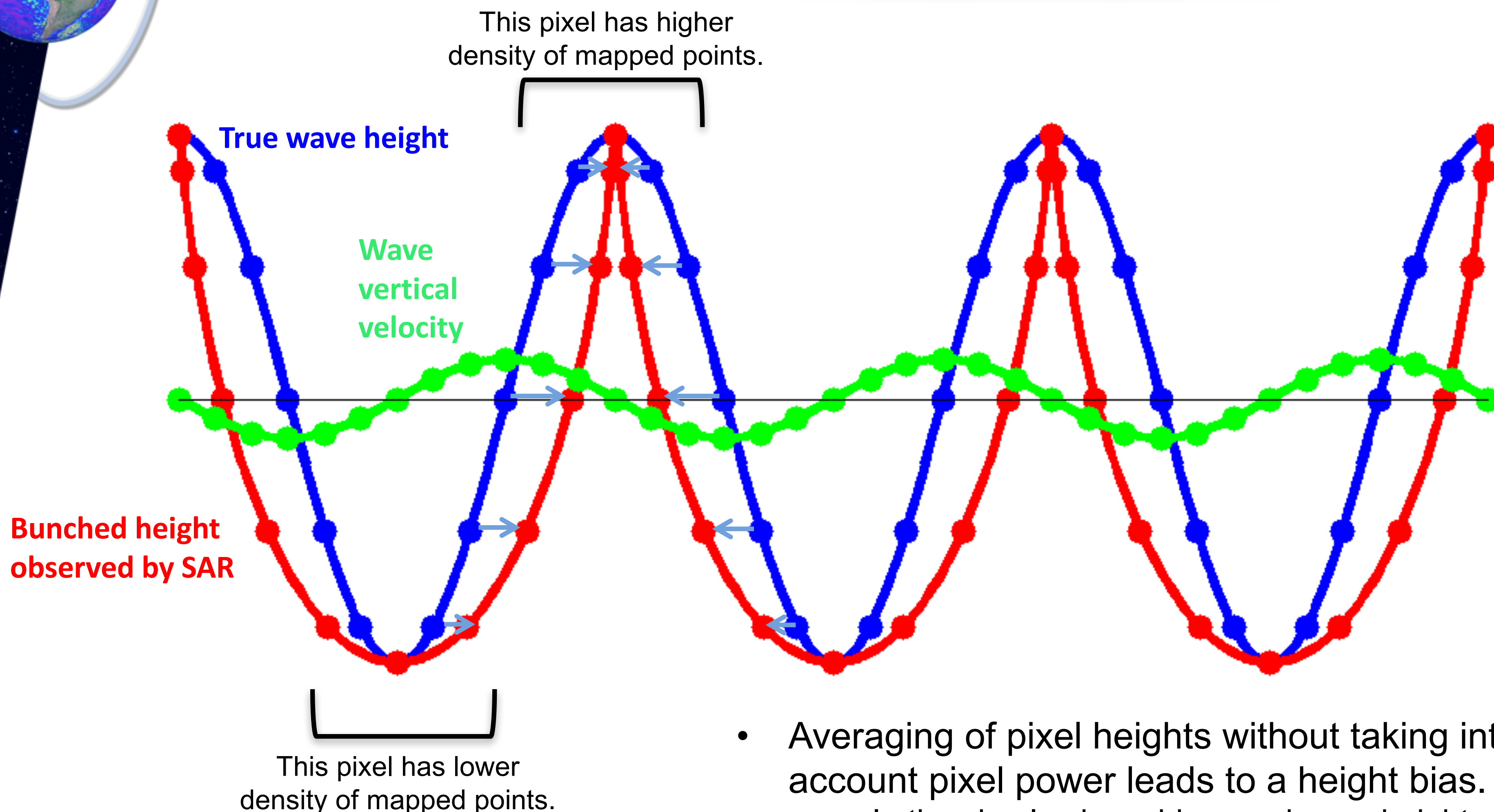
PSD allocations for cross-track and radial velocities

- We consider that attitude errors are fixed right at the value of the requirement level, to derive an upper bound of the error that can be used to generate an allocation.
- In addition, it is also worth noting that the errors associated to yaw are second-order compared to those induced by a pitch.

$$E_{surface\ velocity}(f) = \left[\left(\bar{C} \frac{\alpha_{pitch}}{v_p} \right)^2 + \left(\frac{\bar{C}^2}{H} \frac{\alpha_{yaw}}{v_p} \right)^2 \right] E_{v_s}(f)$$

$$E_{vertical\ velocity}(f) = \left[\left(H \frac{\alpha_{pitch}}{v_p} \right)^2 + \left(\bar{C} \frac{\alpha_{yaw}}{v_p} \right)^2 \right] E_{v_r}(f)$$

Height Distortion From Wave Bunching



Azimuth shift is proportional to line-of-sight target velocity, which is mainly due to wave vertical velocity for near-nadir viewing geometry

- Averaging of pixel heights without taking into account pixel power leads to a height bias.
  - In the simple sinusoid case shown, heights would be biased low.
- **Wave bunching is a non-linear distortion, so spectrum of observed heights can exhibit energy at spatial frequencies that are not present in the true wave field**

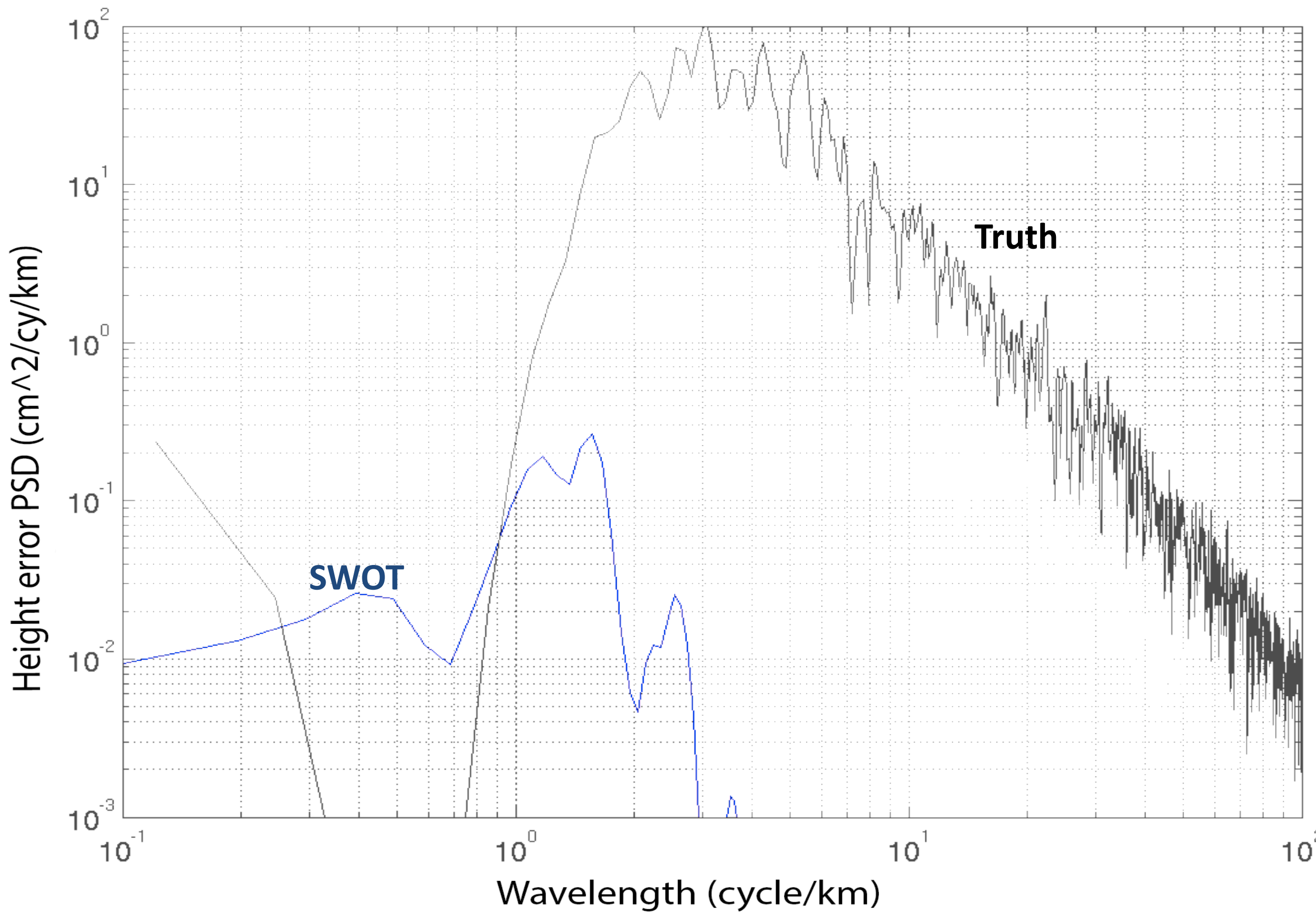
Analytical Model

- Effect of wave bunching has been derived analytically (Rodriguez, et al.), assuming low spatial frequency waves (long wavelength) and low SWH
- Simplified model expression is given by:

$$S_M(k_x, k_y) = \left|\tilde{f}_x(k_x)\tilde{f}_y(k_y)\right|^2 \left[S_h + \left(2k^2 \left|i\frac{r}{v_p}\sin\phi\right|^2 \left(|\omega|^2 S_h\right)\right) * S_h\right]$$

- Measured height spectrum is distorted by a term proportional to the convolution of the radial velocity spectrum and the true height spectrum.
  - This term introduces leakage into the lower range of along-track frequencies, producing a spectral “hump” (an unexpected increase in energy within a certain range of frequencies in the measured SSH PSD).
  - Maximum distortion occurs when the waves are perfectly aligned in the along-track direction (ϕ=90 deg), and effectively vanishes when they are completely aligned in the cross-track direction (ϕ=0 deg).
- For strong non-linearities, linearization approximation of analytical model breaks down
  - Analytical model provides basic intuition and validation of simulations for weakly nonlinear cases
  - Simulations are used to capture non-linearities without approximations at larger SWH and/or shorter wavelength cases

Worst-Case 2 m SWH Case



Wave bunching simulation for 2.8 cy/km waves oriented along-track for 2m SWH.
The black line represents the true spectrum of the waves, and the blue line the resulting wave bunching error for SWOT considering the 500 x 500 m averaging performed by the on-board processor, and additional cross-track unweighted height averaging to 3.5 km.

Differences Between SWOT and AirSWOT

- Relevant differences between SWOT and AirSWOT
  - SWOT OBP does averaging on complex interferogram values to 500 x 500 m resolution; AirSWOT averages high-resolution height samples without power weighting

SWOT OBP

$$h = C\angle\left(\sum_k A_k \exp(j\phi_k)\right)$$

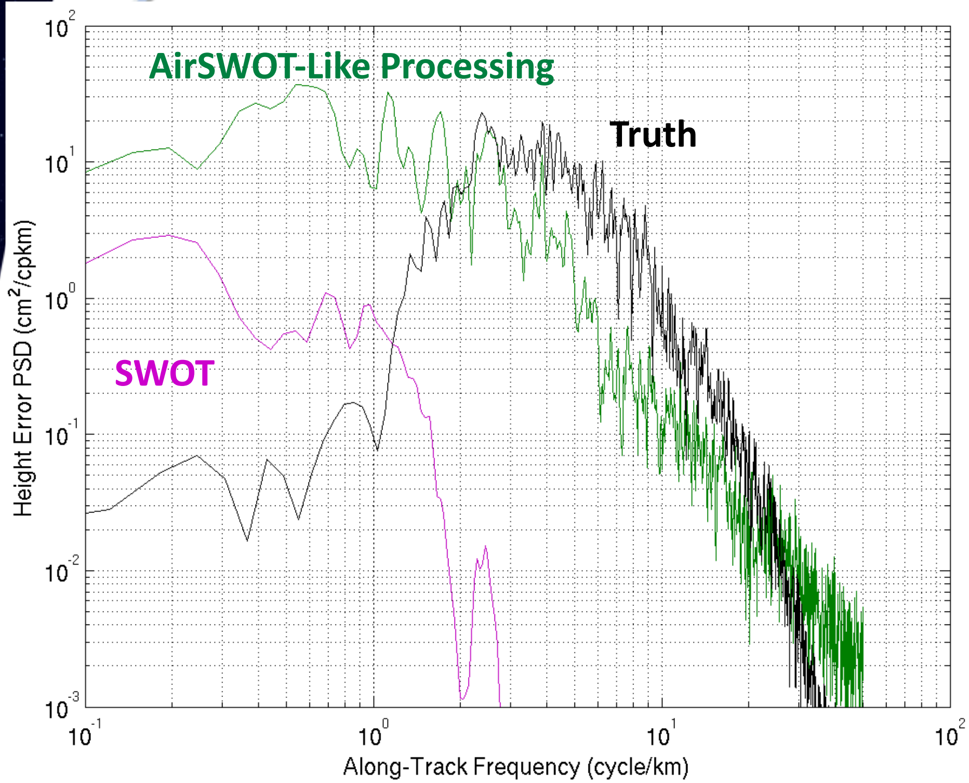
AirSWOT Processor

$$h = \sum_k C\angle\left(A_k \exp(j\phi_k)\right)$$

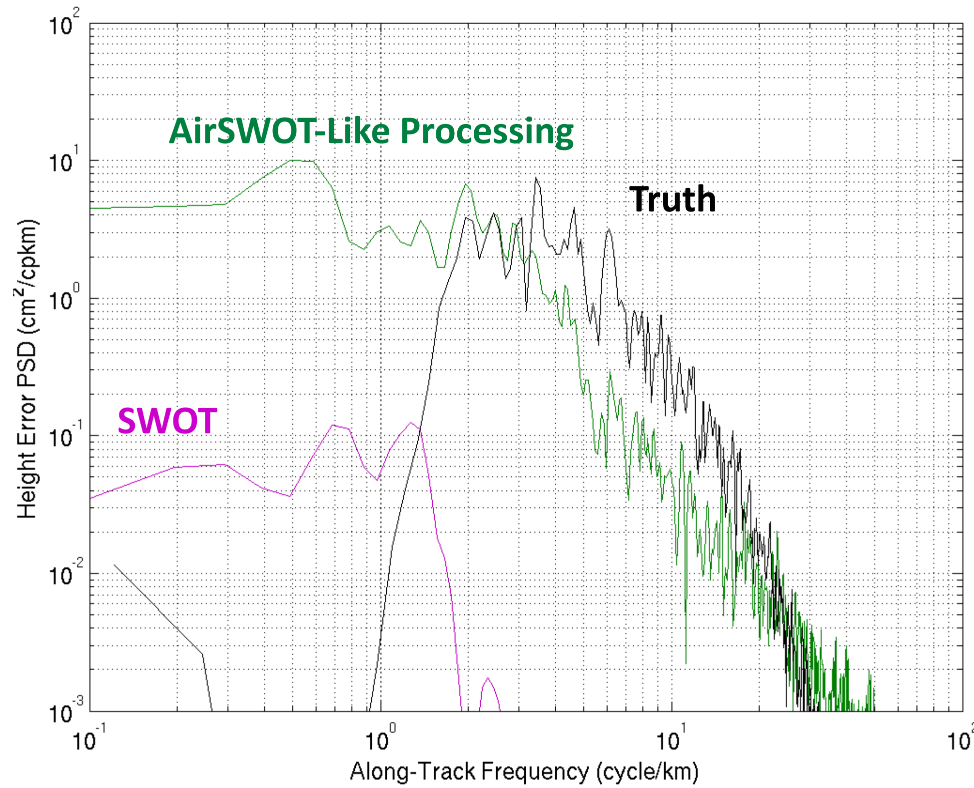
  - SWOT has much wider swath and can do more cross-track averaging (L2 requirement assumes 7.5 km); AirSWOT cross-track averaging is limited to ~1–2 km
  - AirSWOT airborne altitude implies additional distortion as cross-track sample spacing changes with incidence angle over length scales of ocean waves
  - AirSWOT is more sensitive to height errors from cross-track-shift/phase-bias mechanism (large pitch, lower platform velocity), which cannot be corrected when wave bunching corrupts ATI velocity estimate
- **SWOT differences from AirSWOT imply lower sensitivity to height errors from wave-bunching mechanism**

Simulated AirSWOT vs. SWOT Spectra with AirSWOT 20150417 Input Spectrum

SWH = 3 m for these simulations



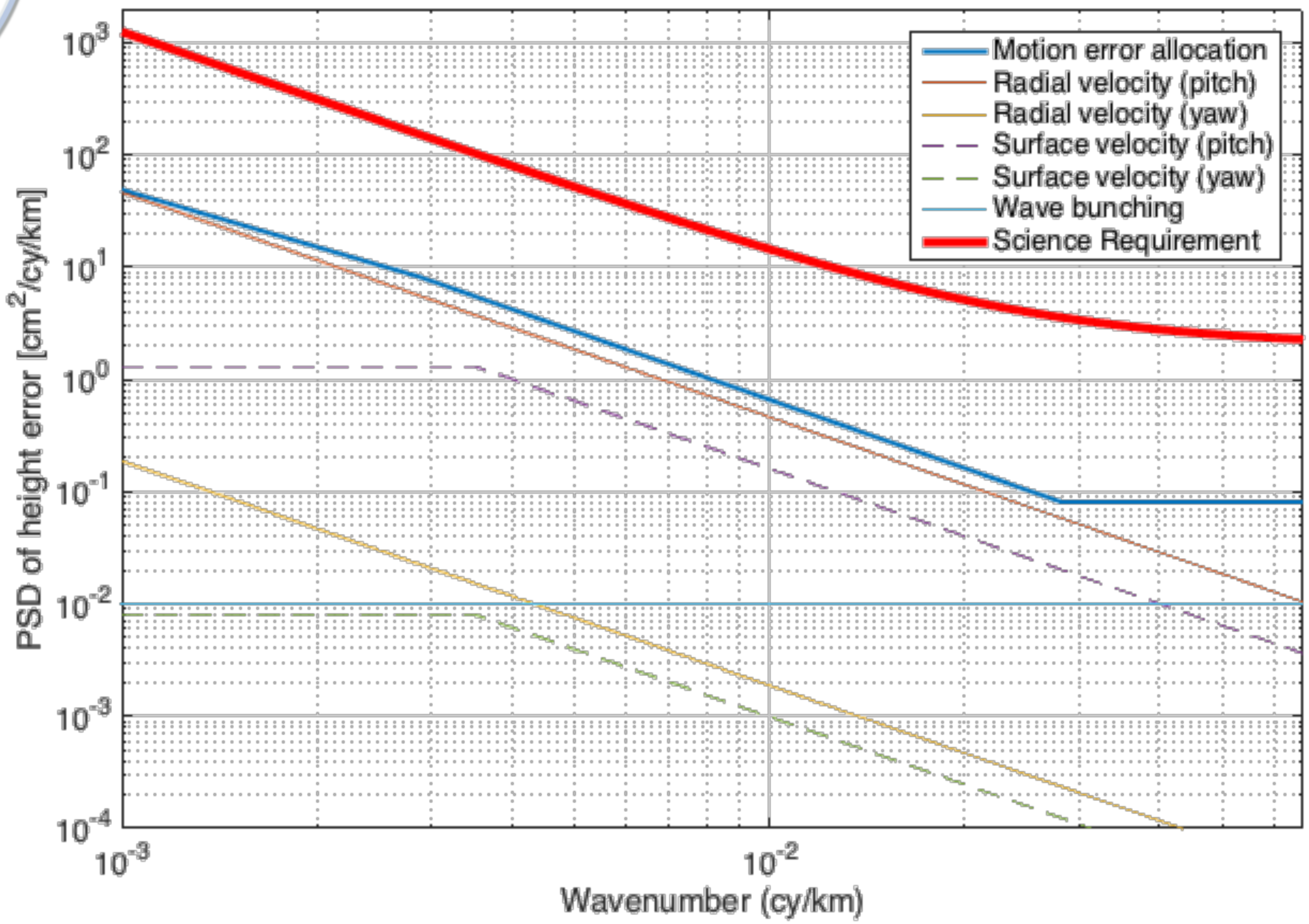
(500 m x 500 m) interferogram average, then **1 km** cross track unweighted height average



(500 m x 500 m) interferogram average, then **3.5 km** cross track unweighted height average

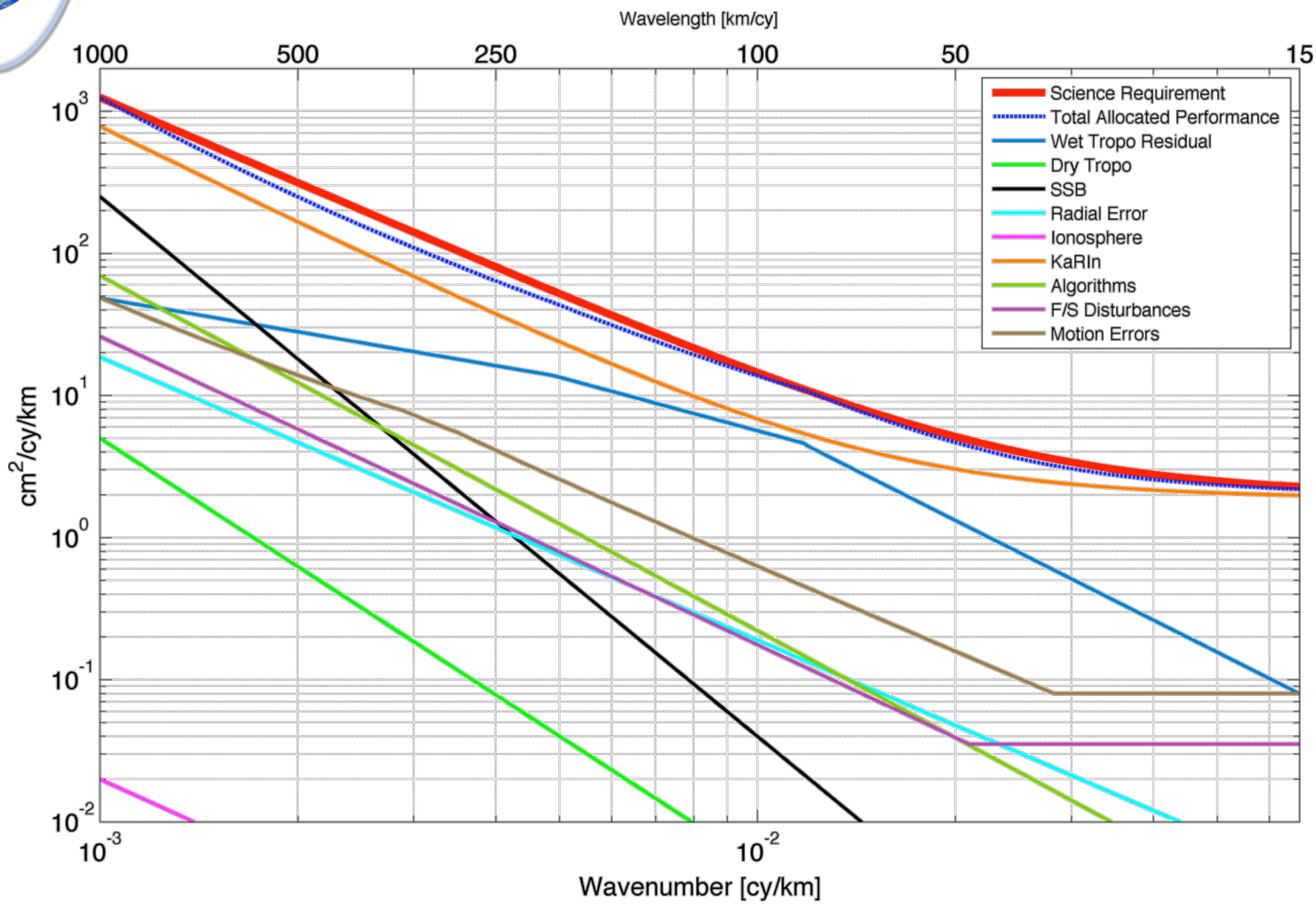
SWOT is much less sensitive to wave-bunching errors than AirSWOT

SWOT Motion error allocations



Note these allocations assume motion error PSDs at the full 1-sigma pointing error requirement, 3% EM bias & 240 deg modulation phase.

Ocean Top-Level Error Budget Allocations



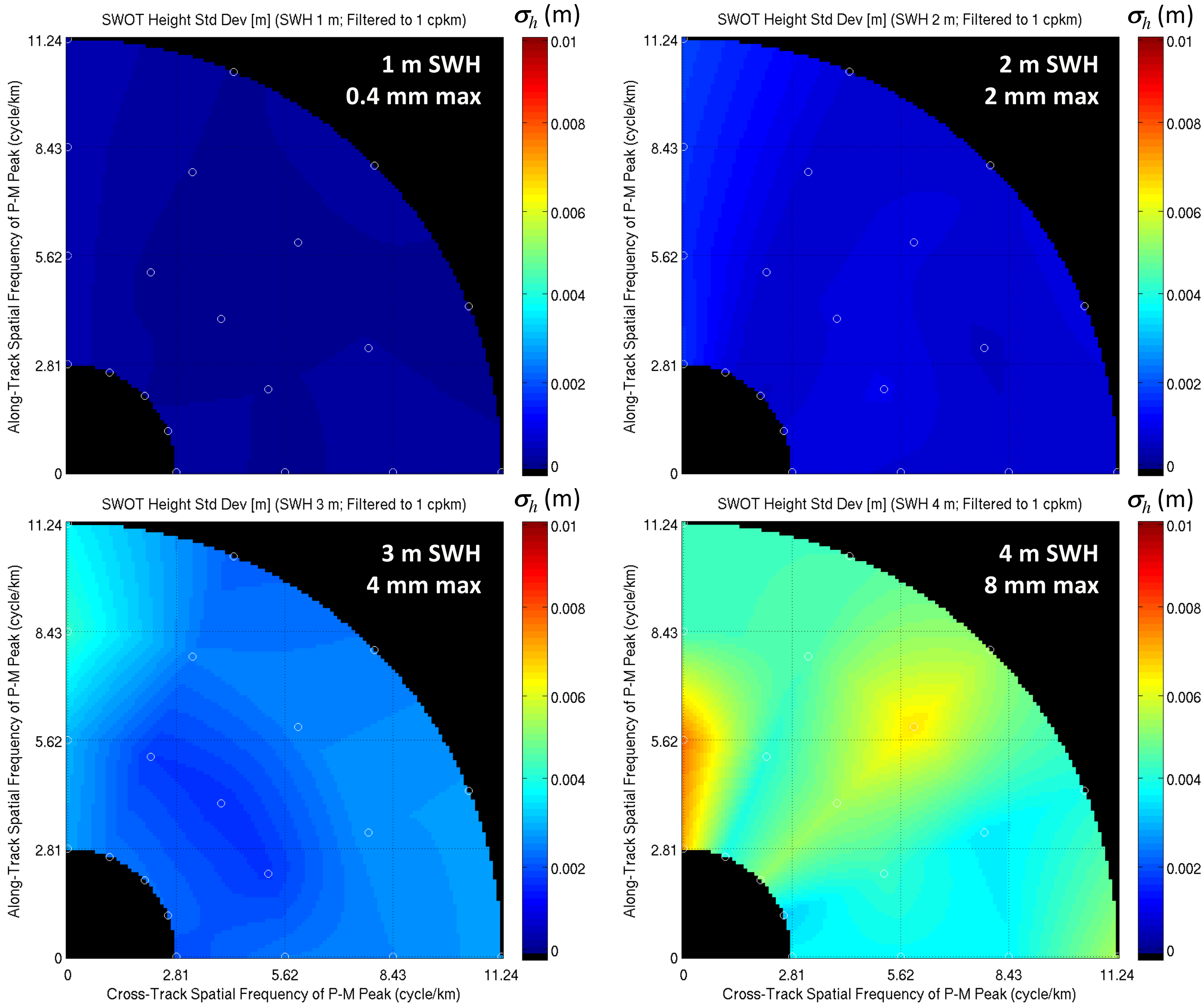
Summary

- The effect of motion errors on the SWOT performance is well understood.
  - Errors with perfect attitude are negligible; only become relevant under the presence of attitude control errors.
  - With the existing attitude control requirements, the error is small
  - Wave bunching is a small effect due to averaging at complex interferogram level on-board and cross-track averaging.
- Motion errors are now formally accounted for in the error budget document.
  - The error contribution is small and can be absorbed with small impact to existing margins.
  - Conservative envelope of error over the oceans is ~one order of magnitude lower than the SSH spectrum for Science Requirements specifications (2m SWH).

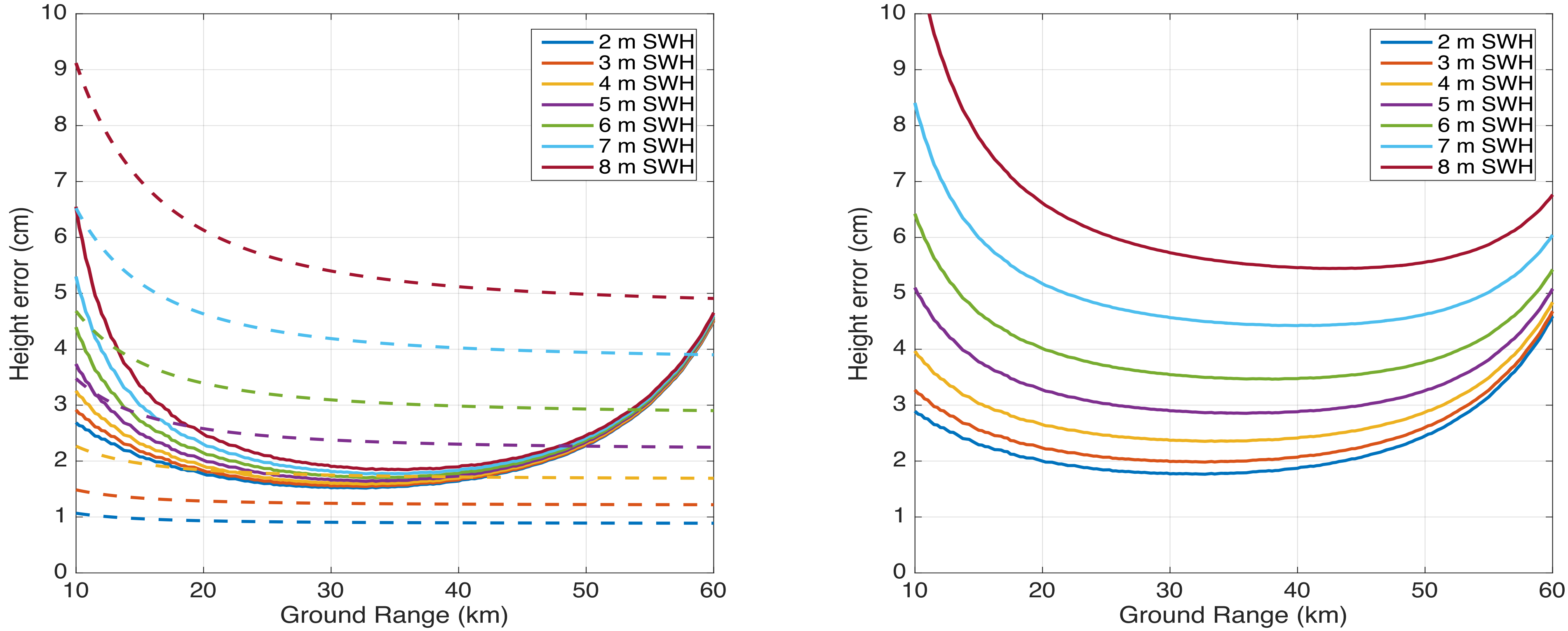
Backup

Simulated SWOT Height Std Dev From Wave Bunching

- Results plotted vs. P-M peak location in spatial frequency plane
- Results interpolated between sim points (white circles)
- Errors are smaller than equivalent integrated SWOT reqt of 1.4 cm for SWH < 2 m

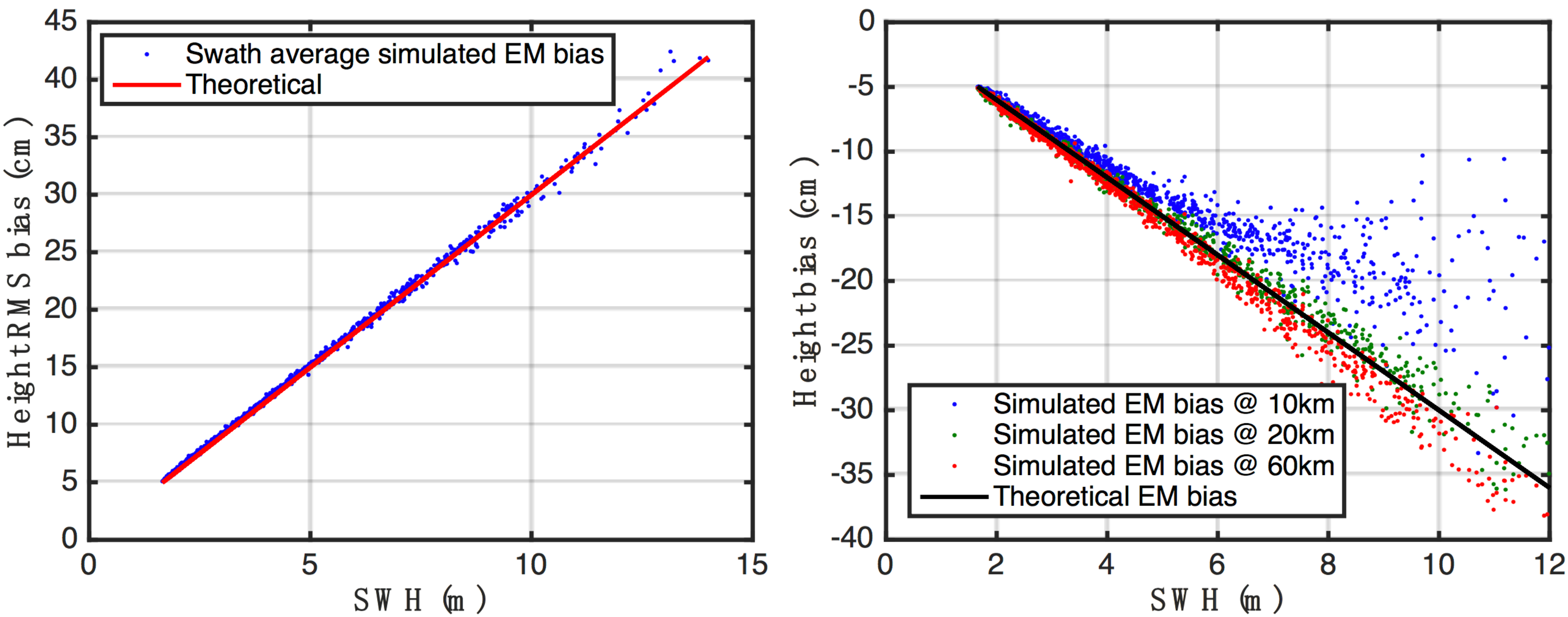


Surf board



2 m SWH
3 m SWH
4 m SWH
5 m SWH
6 m SWH
7 m SWH
8 m SWH
Height error (cm)
Ground Range (km)
2 m SWH
3 m SWH
4 m SWH
5 m SWH
6 m SWH
7 m SWH
8 m SWH
Height error (cm)
Ground Range (km)

EM bias



Swath average simulated EM bias
Theoretical
Height RMS bias (cm)
SWH (m)
Simulated EM bias @ 10km
Simulated EM bias @ 20km
Simulated EM bias @ 60km
Theoretical EM bias
Height bias (cm)
SWH (m)