


# On the spectral slope discontinuity in SSH spectra

A background image of a satellite in orbit over the Earth's ocean surface. The satellite has a central body with various instruments and two large rectangular solar panel arrays extending outwards.

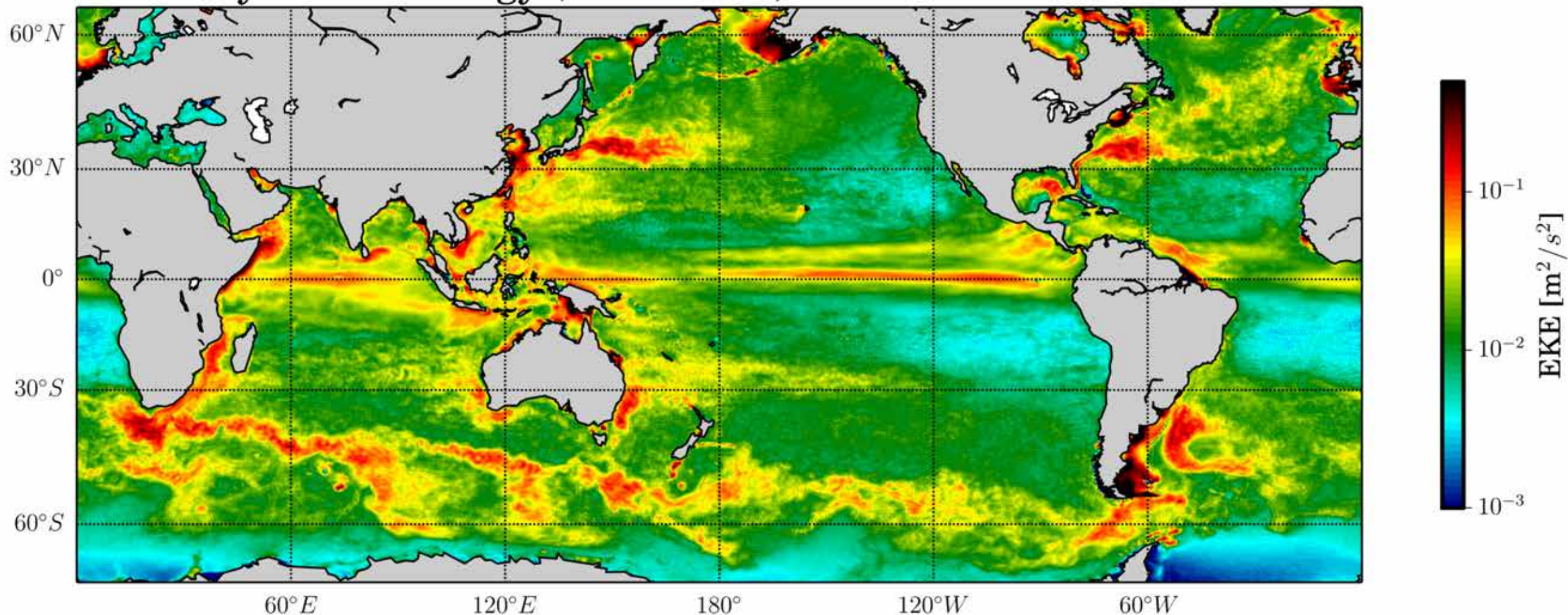
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## Collaborators:

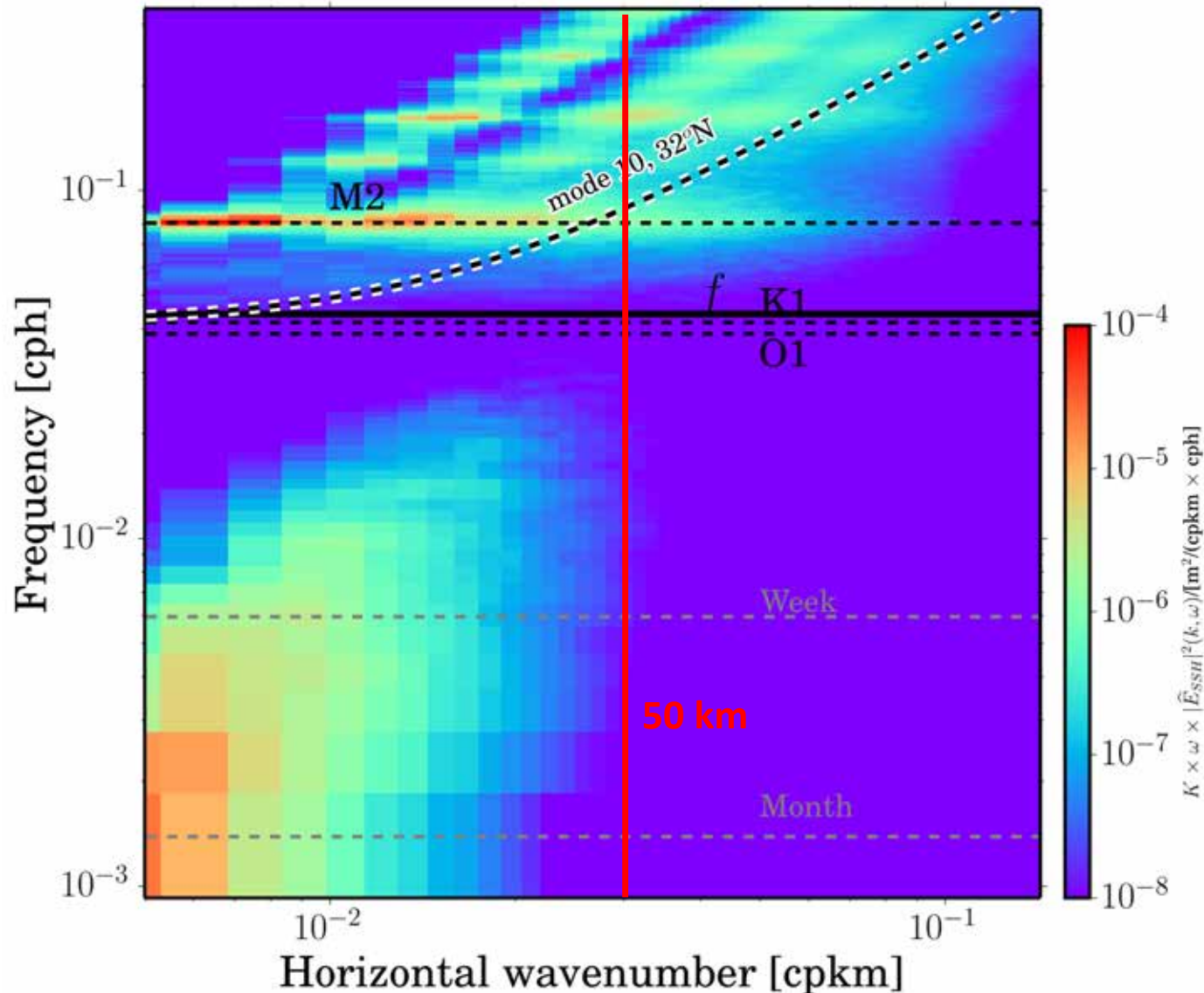
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<sup>1</sup>Jet Propulsion Laboratory/Caltech, USA, <sup>2</sup>LOPS-IFREMER/CNRS France,  
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Surface eddy kinetic energy ( $1/2^\circ \times 1/2^\circ$ ): November 2011 to October 2012



# IGWs dominance at higher wavenumbers in summertime



Frequency-wavenumber spectra  
of SSH: Kuroshio Extension in  
**summertime**

- SSH mostly explained by BMs at scales larger than 50 km
- **SSH mostly explained by IGWs at scales smaller than 50 km**



# Loss of geostrophic balance

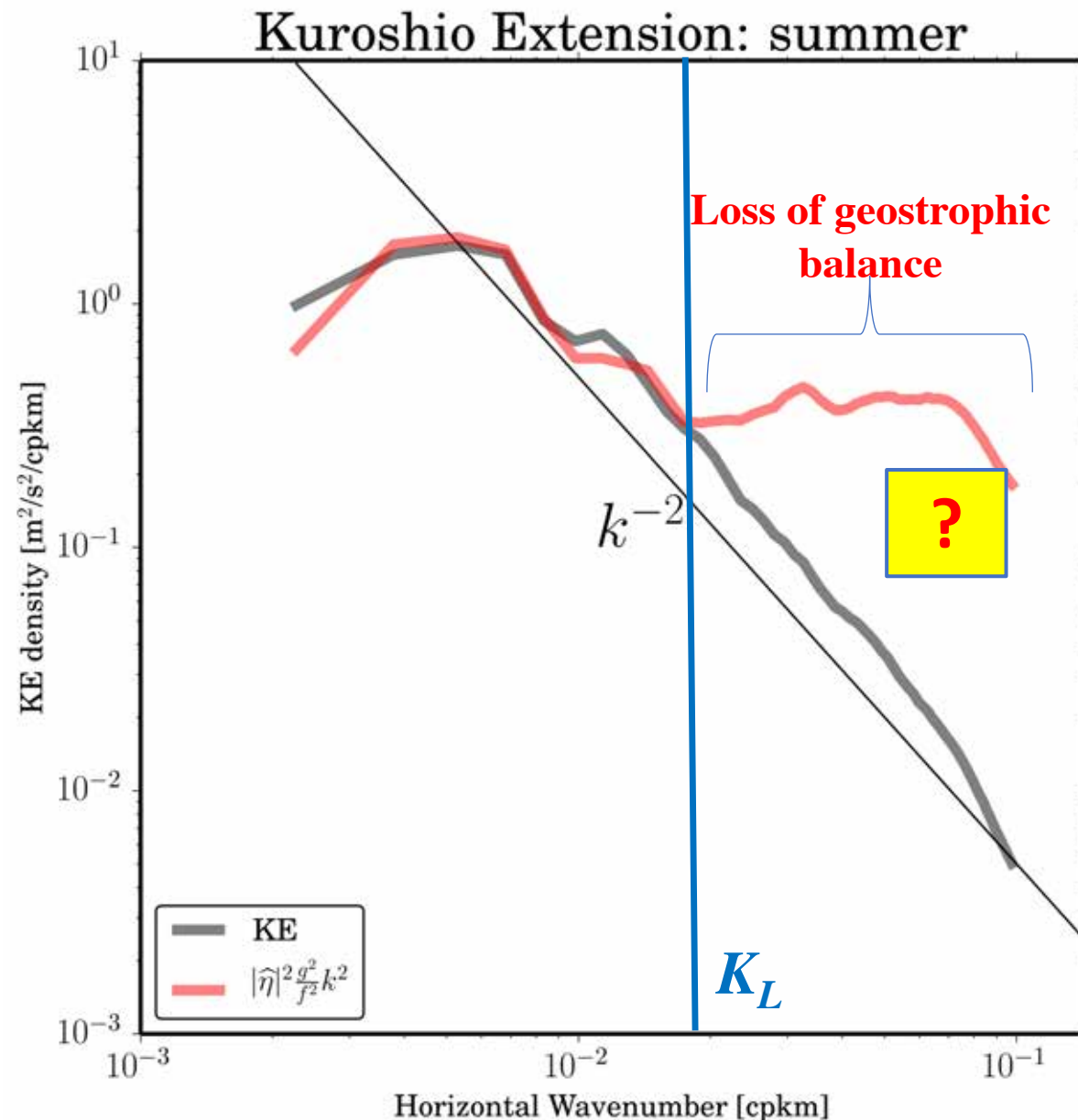
- Geostrophic KE highlights the spectral slope discontinuity

$$\widehat{KE} = |\hat{\eta}|^2 \frac{g^2}{f^2} k^2$$

$K_L$  is the transition scale (Qiu et al. 2018)

See Oscar's talk, for observational evidence

- KE spectra do not display any discontinuity



# Kinetic energy of IGWs

- Linear shallow water approximation is used to recover the continuity of the spectral slope

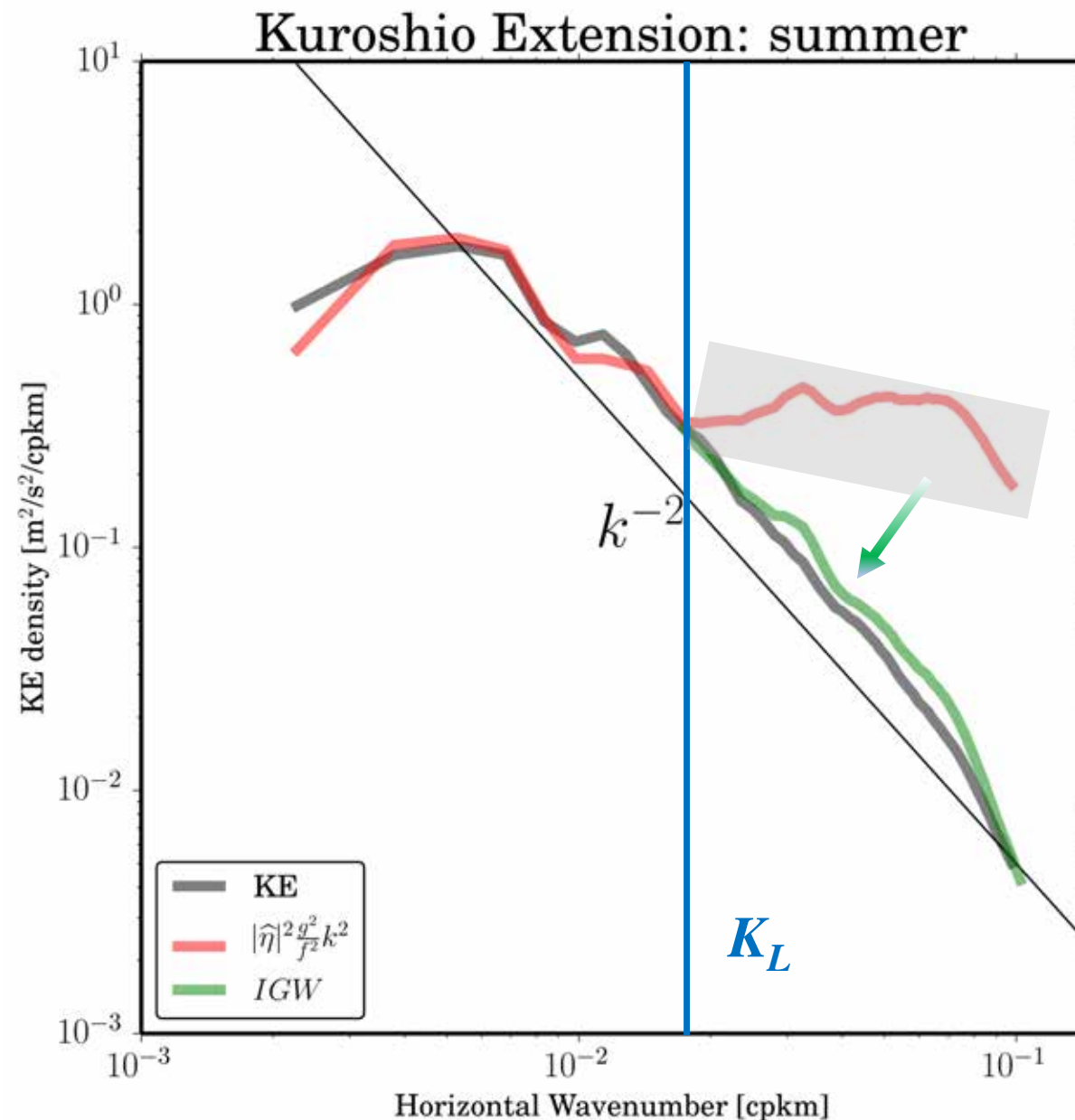
$$\widehat{KE}_{igw} = |\hat{\eta}|^2 g^2 k^2 \frac{(\omega^2 + f^2)}{(\omega^2 - f^2)^2} =$$

$$|\hat{\eta}|^2 \frac{g^2}{f^2} (2 + Rd^2 k^2) / (Rd^4 k^2)$$

$$\omega^2 = f^2 (1 + Rd^2 k^2)$$

$Rd$  is the third baroclinic Rossby radius of deformation

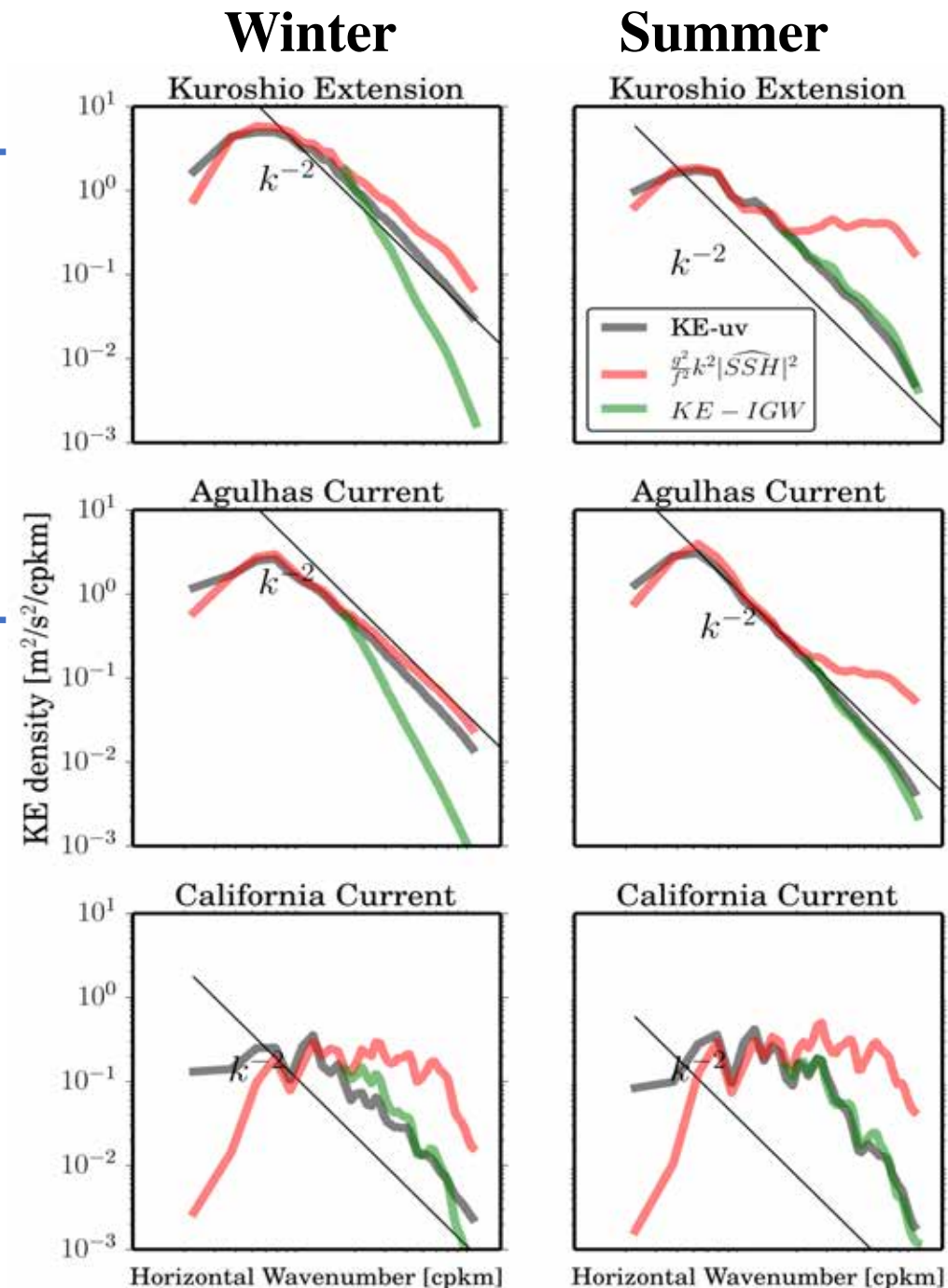
$\widehat{KE}_{igw}$  is the kinetic energy of IGWs



# Spectral slope discontinuity: geographical and seasonal dependence

The methodology works in those high-EKE regions only in summer

The methodology works in those low-EKE regions, either in winter or summer!

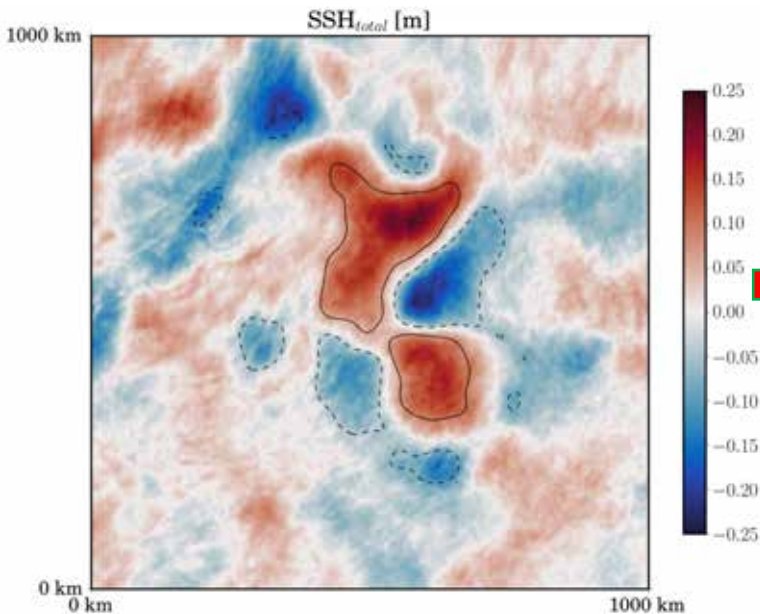


# How to exploit the SWOT wide-swath

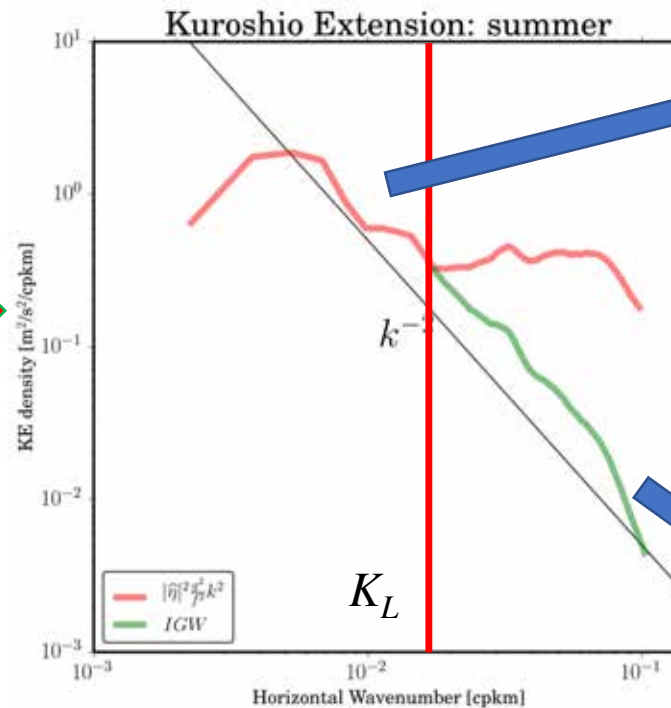
Wide-swath produces  $\eta(x, y)$  and therefore  $\hat{\eta}(k, l)$

Physical space

Physical space



Spectral space



Geostrophic balanced motions:  $\hat{\eta}_{geo}(k, l)$



$$k^2 + l^2 < K_L^2$$

Internal gravity waves:  $\hat{\eta}_{igw}(k, l)$



$$k^2 + l^2 > K_L^2$$

# Reconstructing the velocity field: **Balanced motions + internal gravity waves**

**Geostrophic balanced motions:**

$$\hat{\eta}_{geo}(k, l) \text{ at } k^2 + l^2 < K_L^2$$

$$\hat{u}(k, l) = \frac{igl}{f} \hat{\eta}_{geo}(k, l)$$

$$\hat{v}(k, l) = -\frac{igk}{f} \hat{\eta}_{geo}(k, l)$$

**Internal gravity waves using shallow water equations:**

$$\hat{\eta}_{igw}(k, l) \text{ at } k^2 + l^2 > K_L^2$$

$$\hat{u}(k, l) = g\hat{\eta}_{igw}(k, l) \frac{[\omega k + ilf]}{(\omega^2 - f^2)}$$

$$\hat{v}(k, l) = g\hat{\eta}_{igw}(k, l) \frac{[\omega l - ikf]}{(\omega^2 - f^2)}$$

$$\omega^2 = f^2(1 + Rd_{3rd}^2 K^2)$$

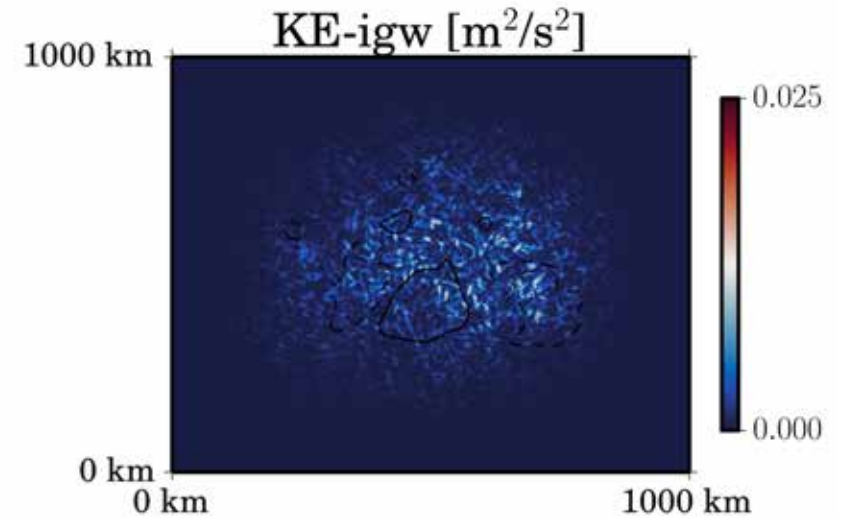
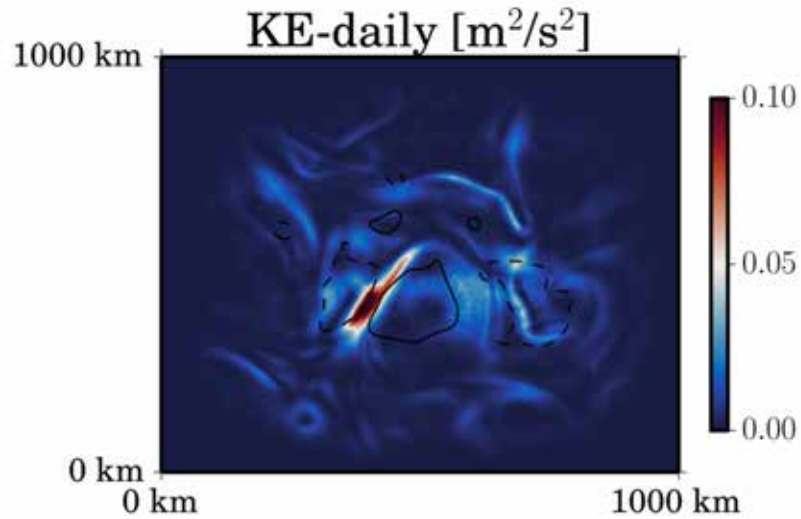
**Inverse FFT**

**Reconstructing the 2D velocity field in the physical space: amplitude and phase**



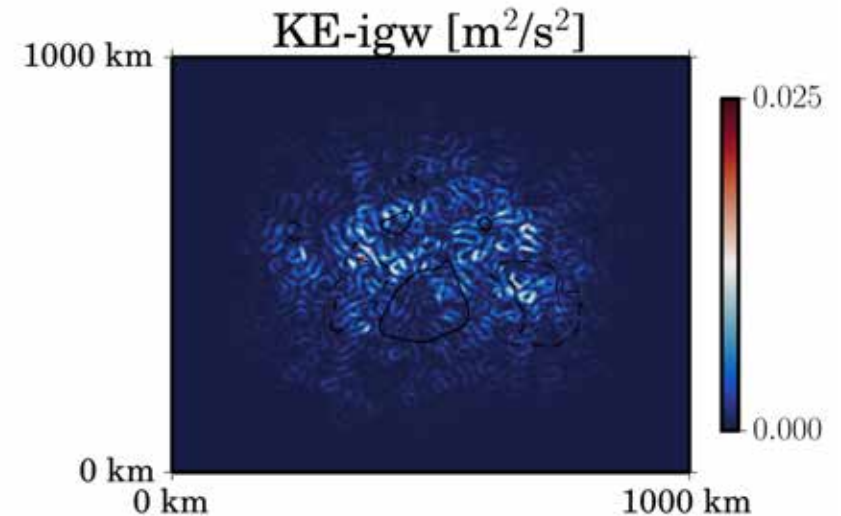
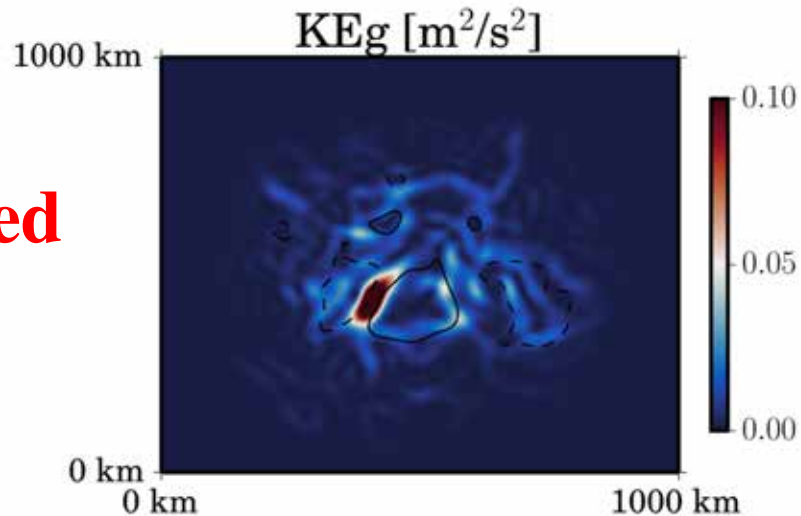
# Comparison between true KE and KE reconstructed

True



Amplitude  
and phase

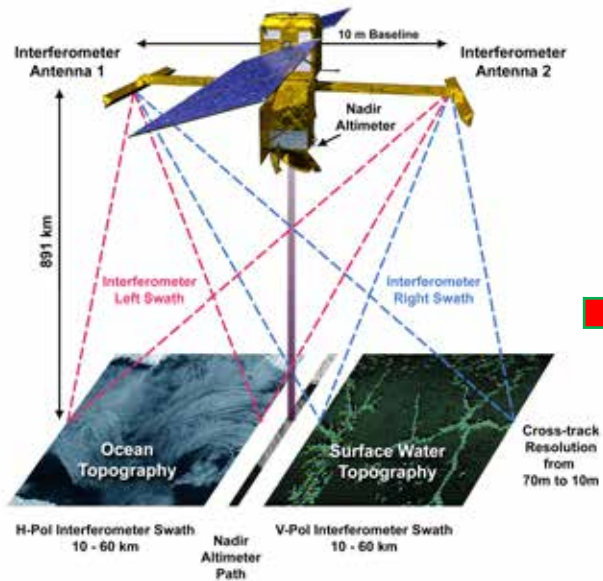
Reconstructed



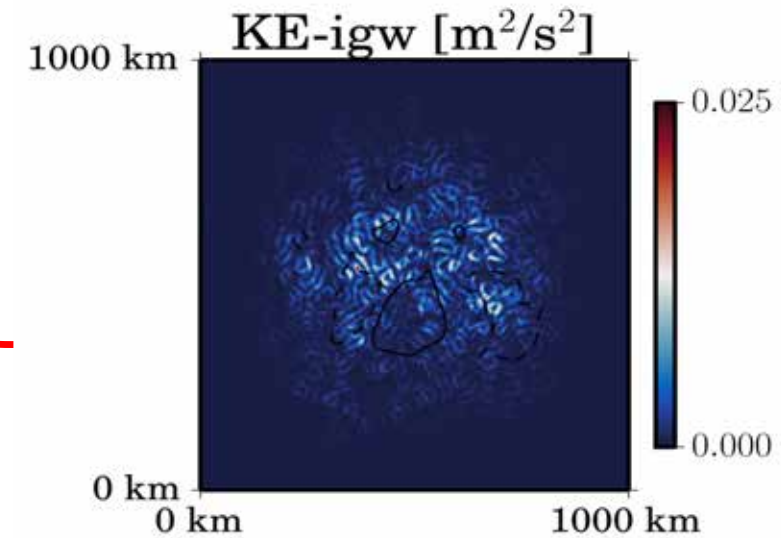
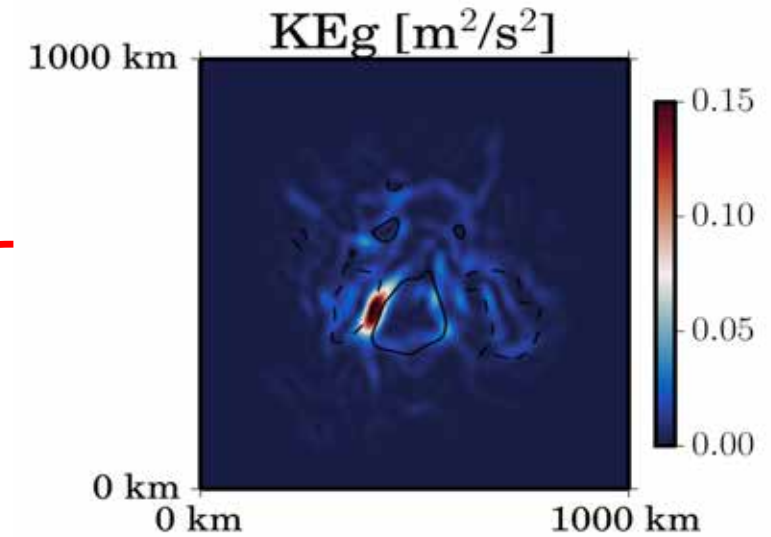
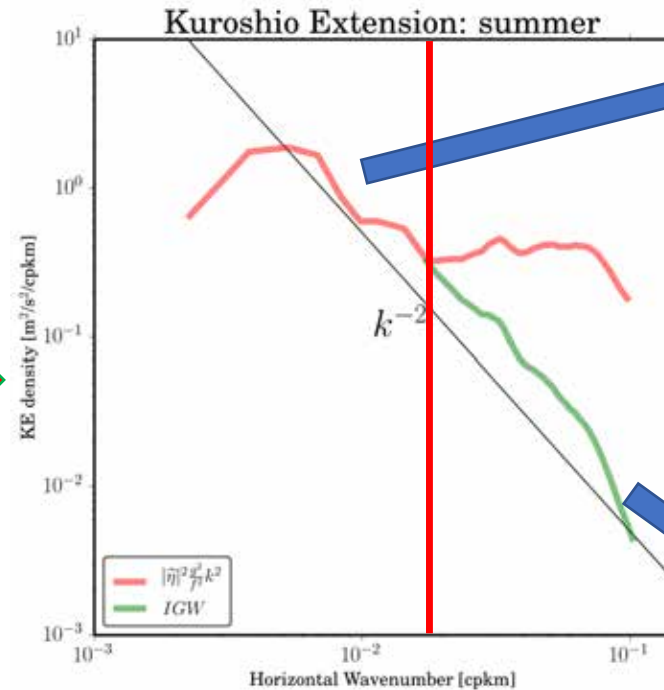
Amplitude  
and phase

# Future applications of the SWOT wide-swath

## SWOT observations



## Spectral analysis



**Such spectral slope discontinuity for SSH, + the wide-swath property of SWOT, can be exploited to diagnose BMs and IGWs !**

# Conclusions

- The reconstructions from SSH( $x, y$ ) over geostrophic balanced motions and IGW is a work in progress
- We would to extend the Kuroshio example to other oceanic regions, in particular the California Current