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Mapping surface tides from SWOT data:

An assessment of regularized and constrained harmonic analysis in the St. Lawrence Estuary

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SWOT 3rd ST Meeting

Montreal, Canada

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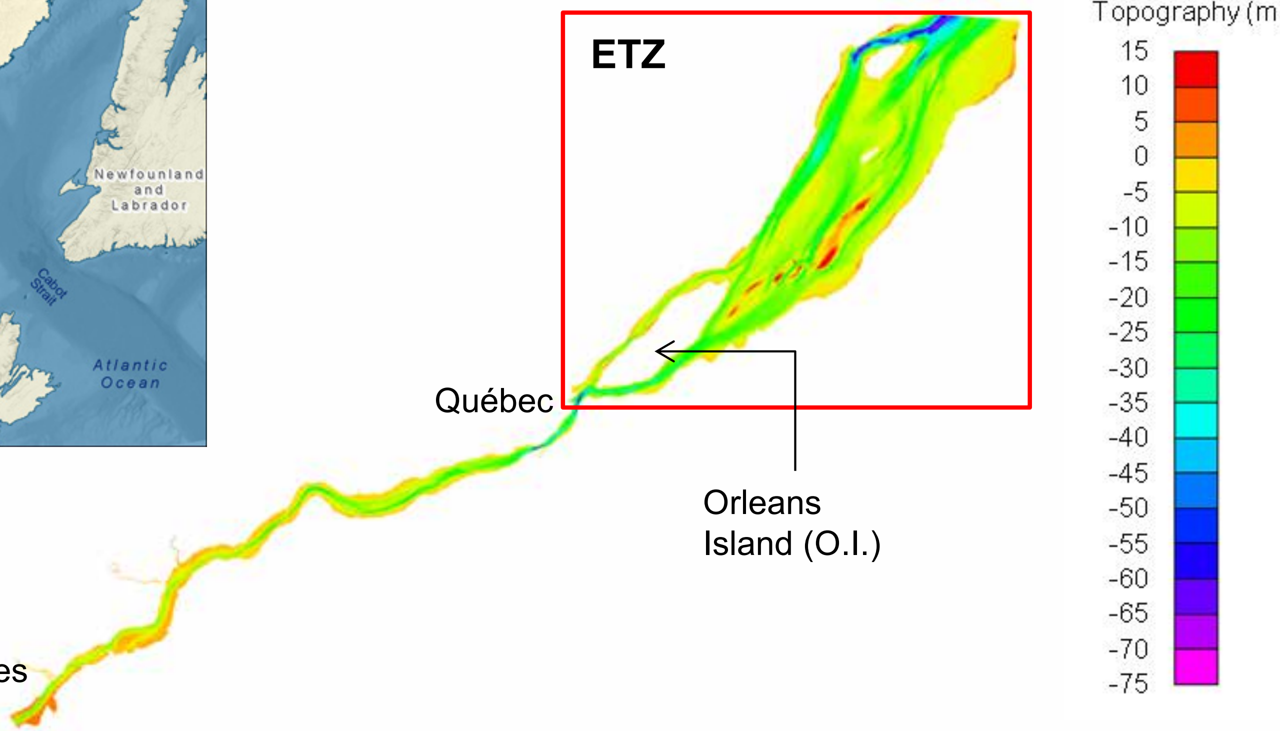
Objectives

- Evaluate the potential of SWOT data to map surface tides in estuaries

- Develop a robust method for the recovery of tidal constituent properties
  - Be robust to SWOT temporal resolution (limited tidal aliasing)
  - Include multiple tidal constituents in each tidal band (good frequency resolution)
  - Allow 2D mapping of nonlinear tides (spatially coherent)
  - Be independent of system geometry (easily transferable)

Study Site

- St. Lawrence Estuarine Transition Zone (ETZ)



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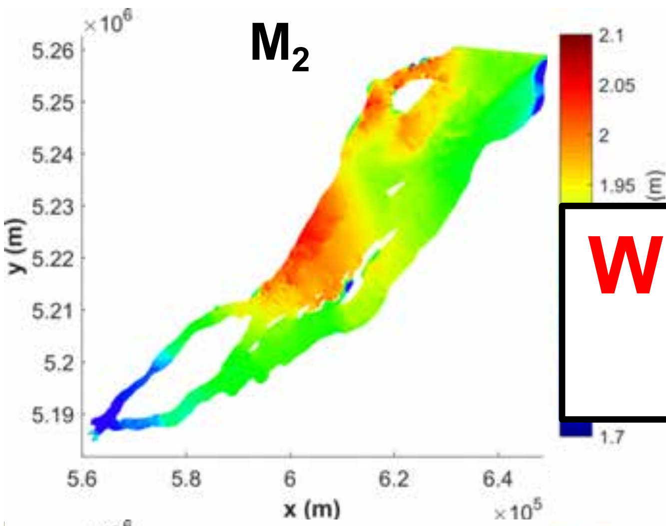
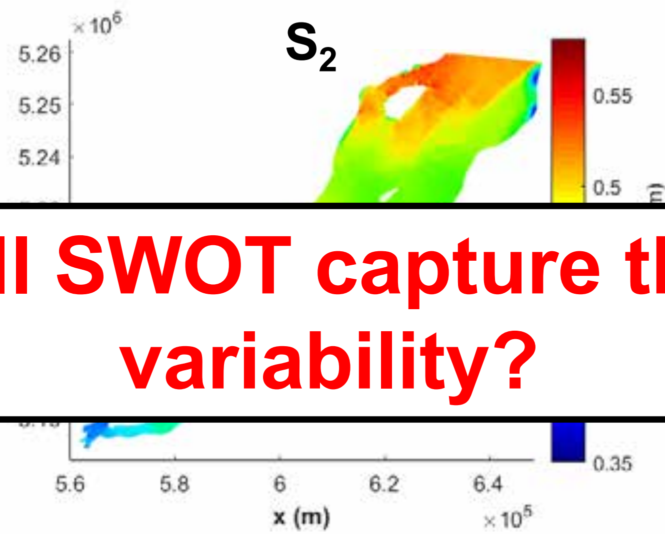
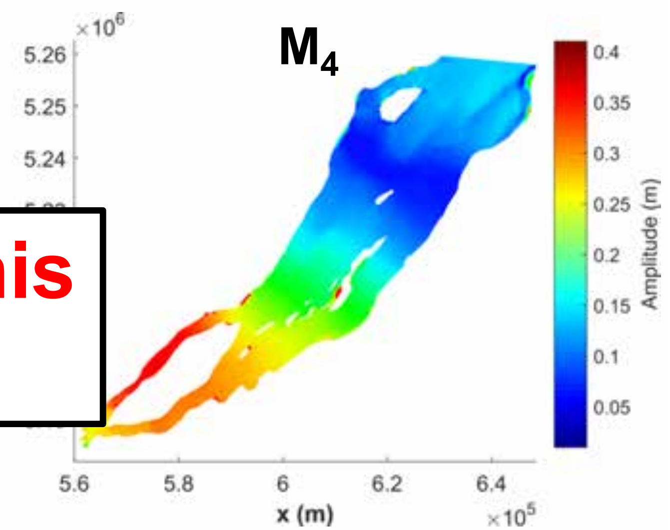
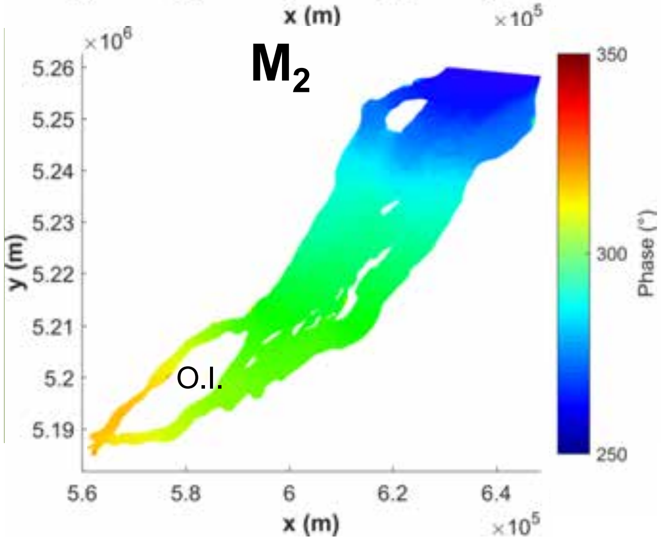






Will SWOT capture this variability?



- Macrotidal (tidal range <7m)
- Lateral variability in tides
- Moving amplitude maximum
- Increasing influence of river discharge upstream
- Asynchronous tidal propagation around O.I.

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Methods

- Classical Harmonic Analysis (HA)
  - Harmonic model:

$$y_c = c_0 + \sum_k C_k \cos(\sigma_k t - \Phi_k)$$

or

$$y_c = c_0 + \sum_k \left[c_{1,k} \cos(\sigma_k t) + c_{2,k} \sin(\sigma_k t)\right]$$

In matrix form: $\mathbf{y} = \mathbf{X}\mathbf{c} + \boldsymbol{\varepsilon}$

y_c:	tidal heights
c_i:	model coefficients
σ_k:	tidal frequency
t:	time
$C_k = \sqrt{c_{1,k}^2 + c_{2,k}^2}$:	amplitude
$\Phi_k = \tan^{-1}(c_{2,k}/c_{1,k})$:	phase

  - Hypotheses:
    - Tides are stationary and independent of time-varying riverine, oceanic and atmospheric influences.
    - There is a fixed number of tidal constituents with discrete periodicities, phase angles and amplitudes.
    - Tidal constituents are mutually independent.

Methods

- Ordinary Least-Squares (OLS)

$$\hat{c} = \arg\min_{c} \left\{ \sum_{i=1}^{n} (y_i - {x_i}^T c)^2 \right\} \text{ or } \hat{c} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y}$$

- Regularized Least-Squares
  - Ridge regression (Hoerl and Kennard, 1970)

$$\hat{c} = \arg\min_{c} \left\{ \sum_{i=1}^{n} (y_i - {x_i}^T c)^2 + \boxed{\lambda} \sum_{j=1}^{p} {c_j}^2 \right\} \text{ or } \hat{c} = \left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right.$$

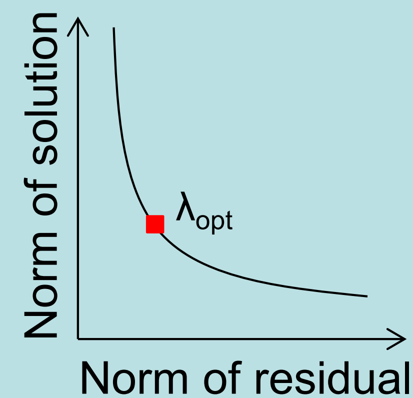
  - Lasso regression (Tibshirani, 1996)

$$\hat{c} = \arg\min_{c_0, c} \left\{ \frac{1}{2n} \sum_{i=1}^{n} (y_i - c_0 - {x_i}^T c)^2 + \boxed{\lambda} \sum_{j=1}^{p} |c_j| \right\}$$

  - Elastic Net regression (Zou and Hastie, 2005)

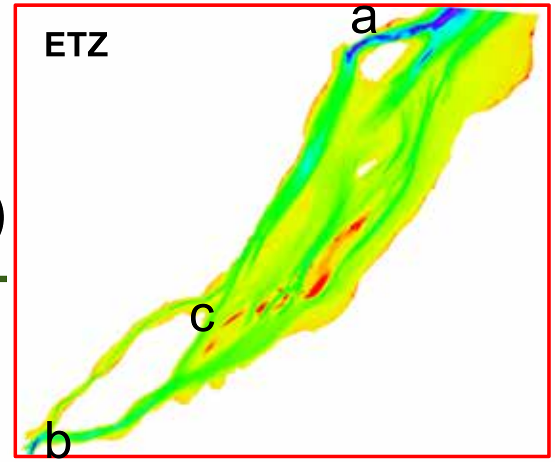
$$\hat{c} = \arg\min_{c_0, c} \left\{ \frac{1}{2n} \sum_{i=1}^{n} (y_i - c_0 - {x_i}^T c)^2 + \boxed{\lambda} \sum_{j=1}^{p} \left(\frac{1-\alpha}{2} {c_j}^2 + \alpha |c_j| \right) \right\}$$

Find optimal λ using *L-curve criterion*



Methods

$$y_a = a_0 + \sum_k A_k \cos(\sigma_k t - \alpha_k)$$
$$y_b = b_0 + \sum_k B_k \cos(\sigma_k t - \beta_k)$$
$$y_c = c_0 + \sum_k C_k \cos(\sigma_k t - \Phi_k)$$

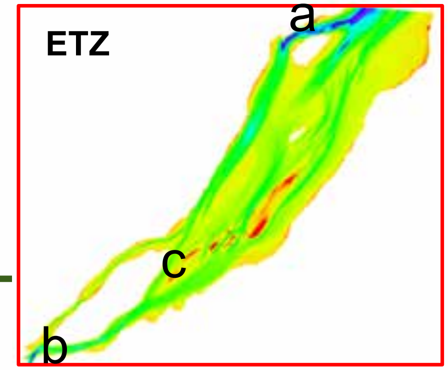


- Constrained Harmonic Analysis (ConHA)
  - Model parameters at location ***c*** are estimated from known tidal constituent properties at two *in-situ* boundary stations ***a*** and ***b***.
  - Constrained minimization:

$$\hat{c} = \arg\min_c \left\{ \sum_{i=1}^{n} (y_i - x_i{}^T c)^2 \right\}$$

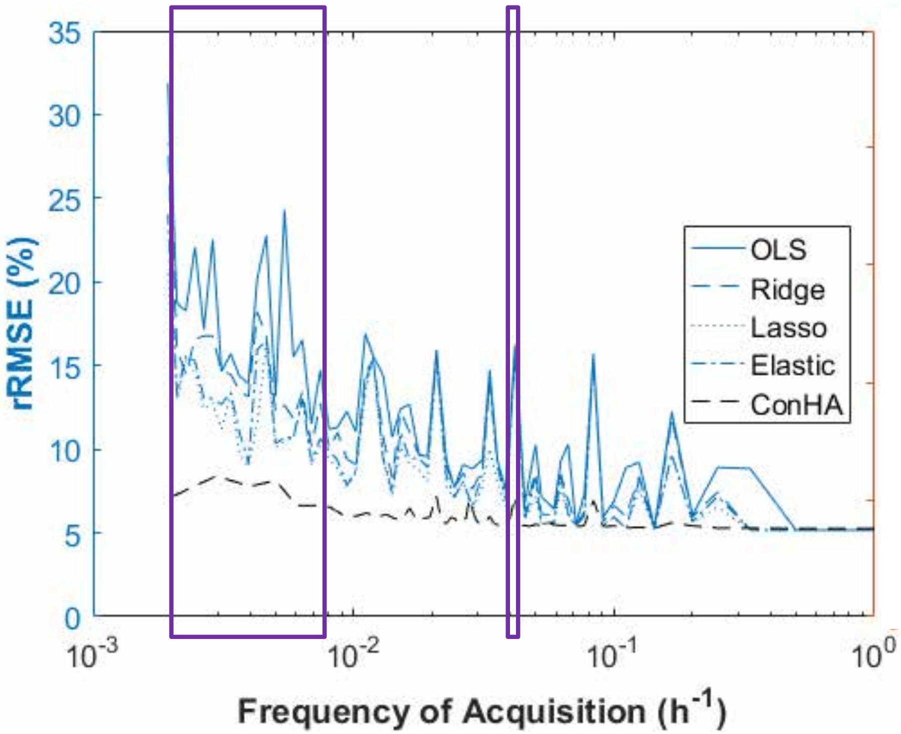
$$\text{subject to} \begin{cases} \min(a_0, b_0) < c_0 < \max(a_0, b_0) \\ (1-f)\times\min(A_k, B_k) < C_k < (1+f)\times\max(A_k, B_k), \\ \alpha_k < \Phi_k < \beta_k \end{cases}$$

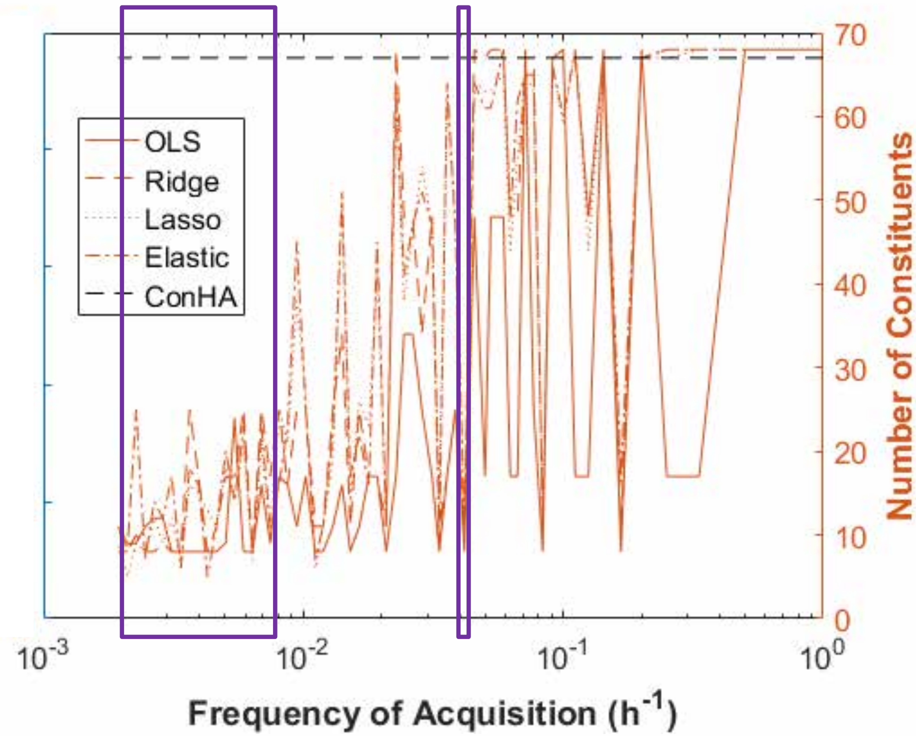
where *f* is an amplification factor.



Sensitivity Analysis

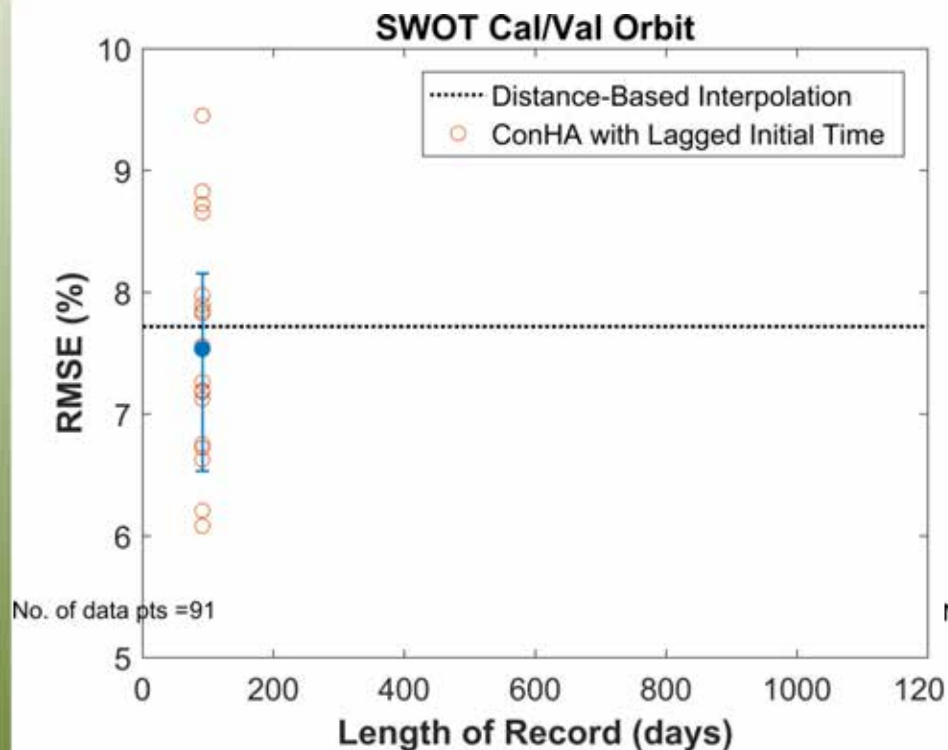
- Sensitivity to frequency of acquisition (1-year signal at station ***c***)

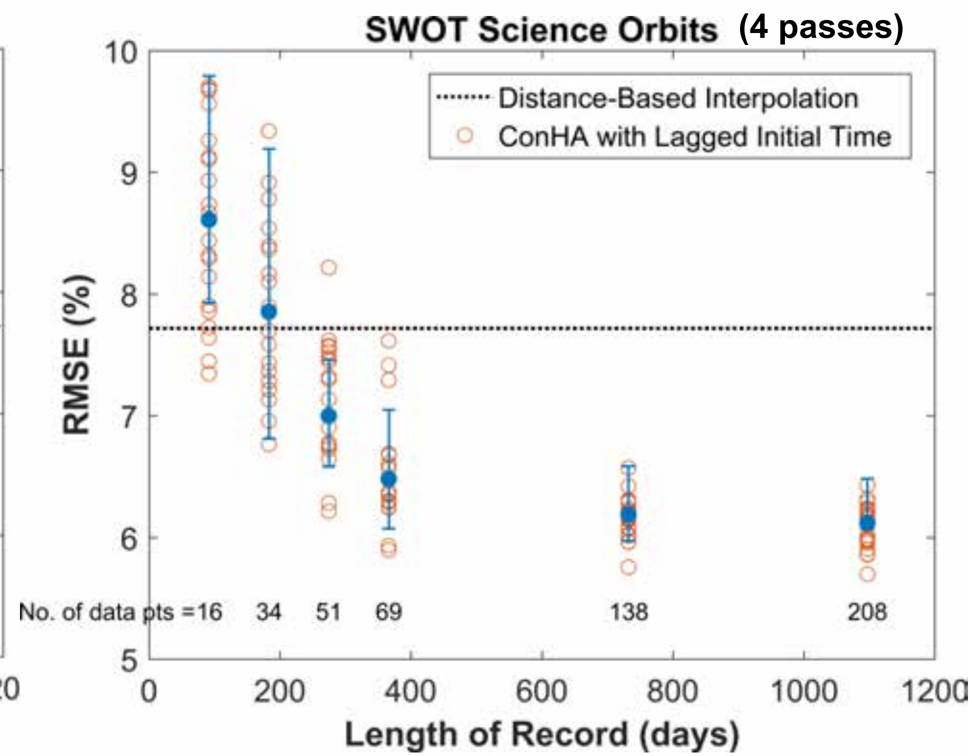




Sensitivity Analysis

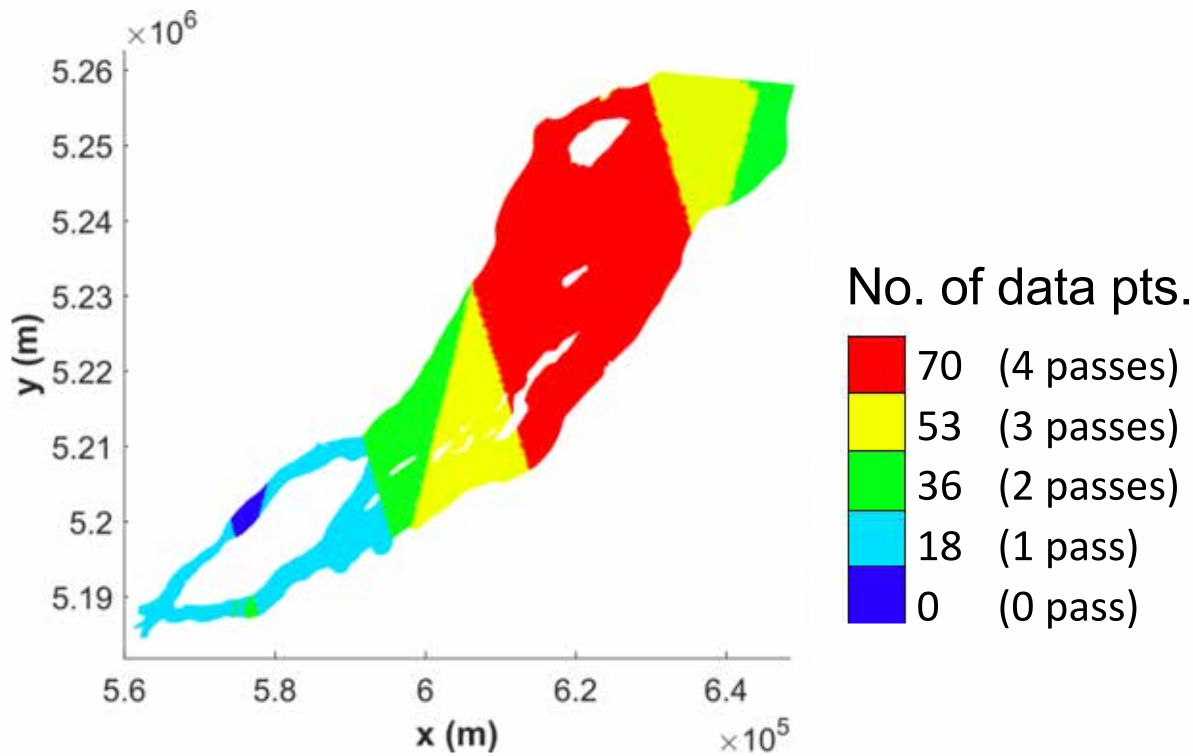
- Sensitivity to SWOT record length and initial launch time



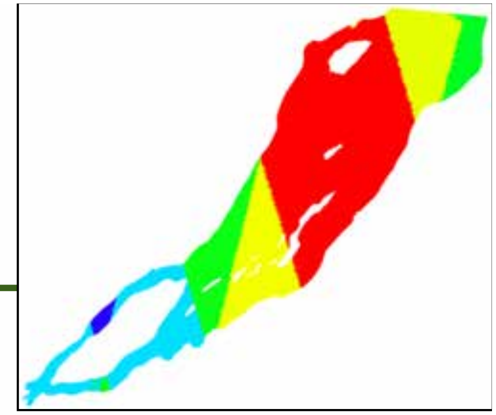


Mapping Estuarine Tides

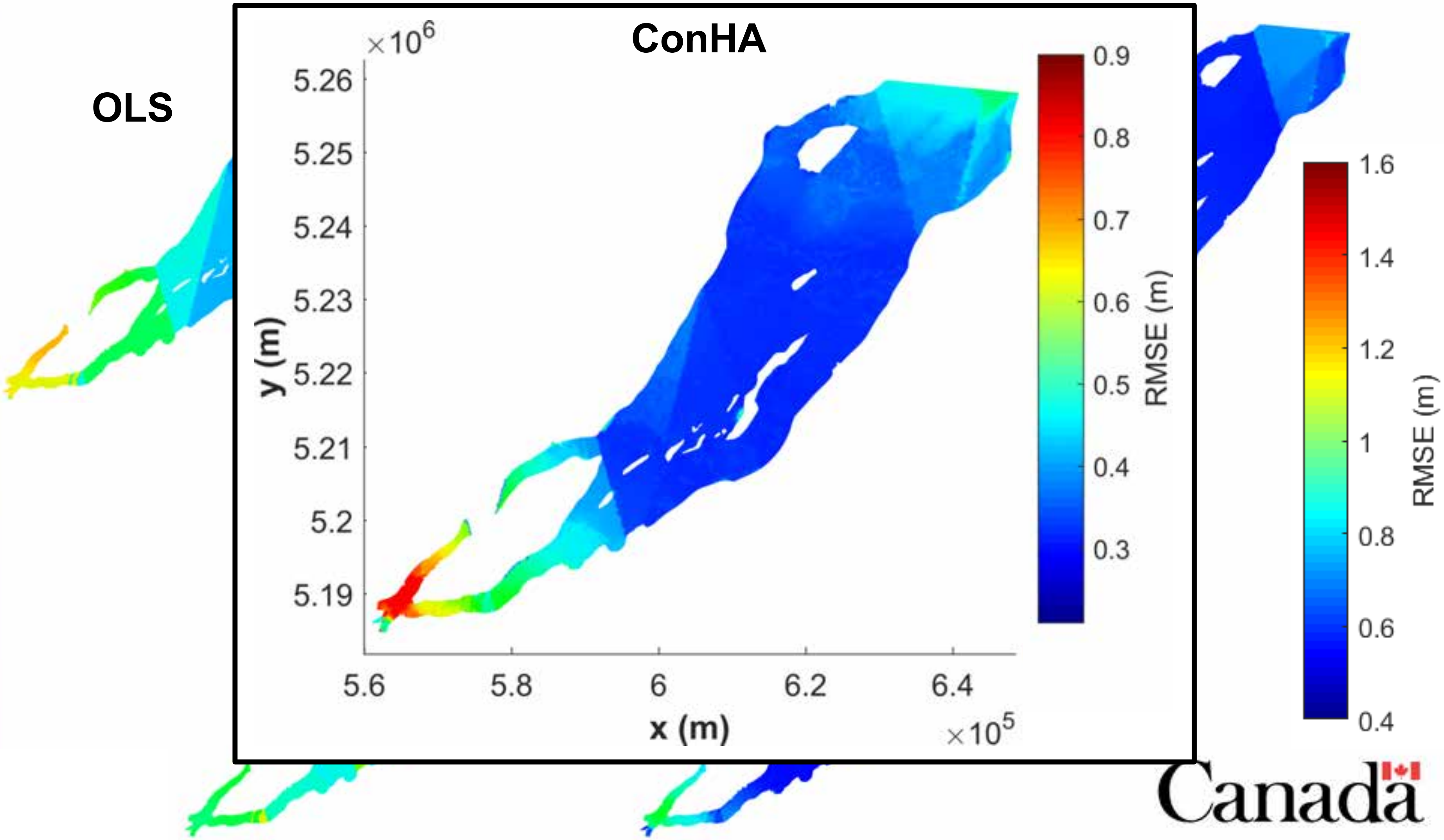
- 2D water level fields from a 1-year numerical simulation were used to produce a 2D SWOT sample



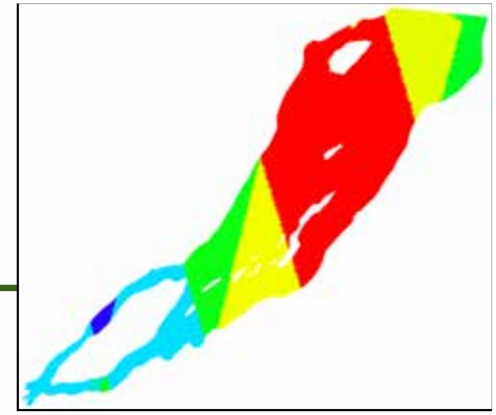
Mapping Estuarine Tides



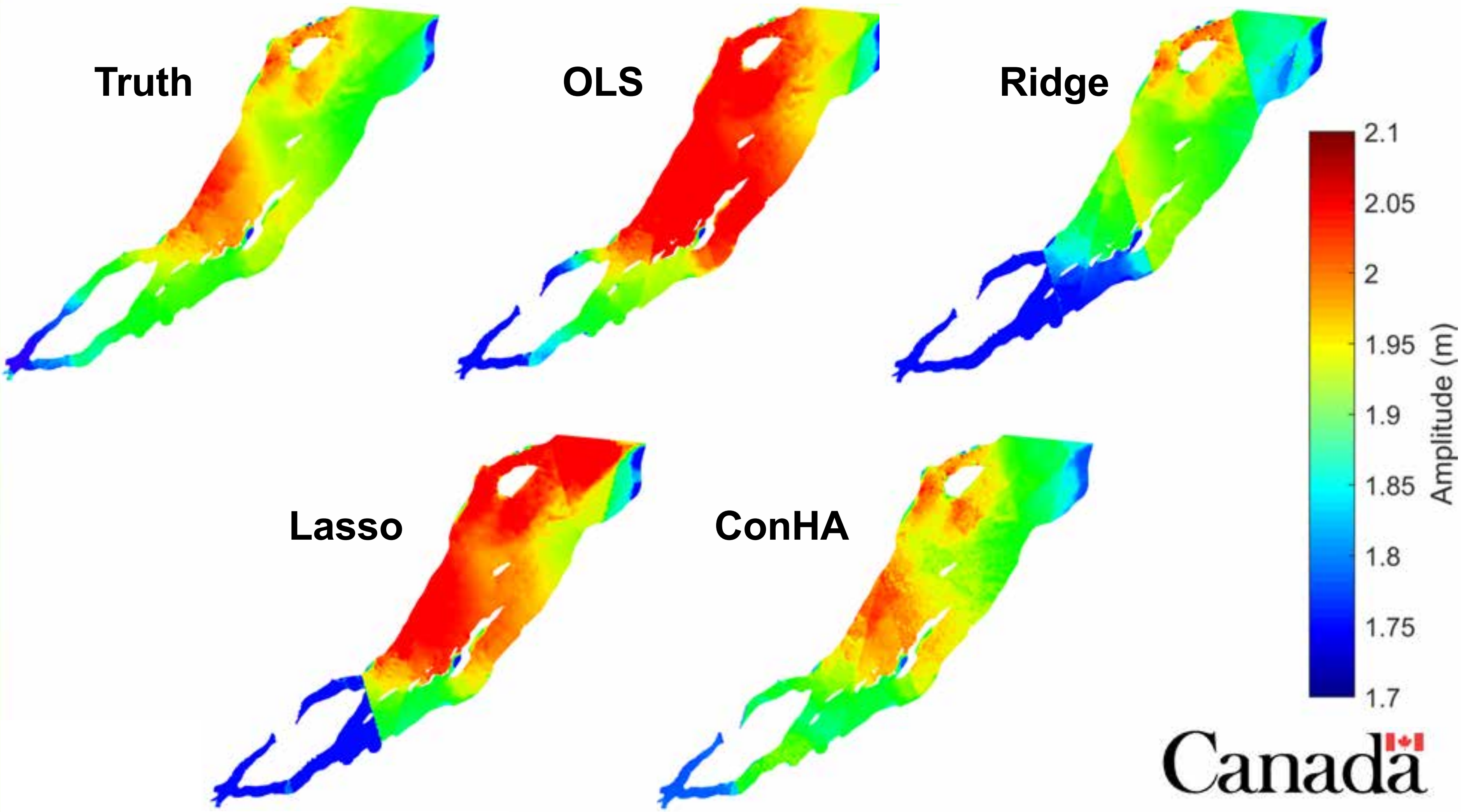
- RMSE



Mapping Estuarine Tides



- M_2 amplitude



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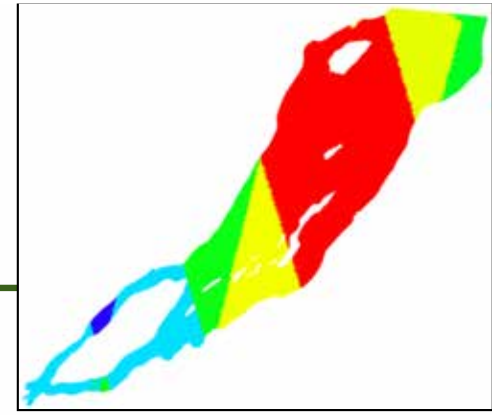
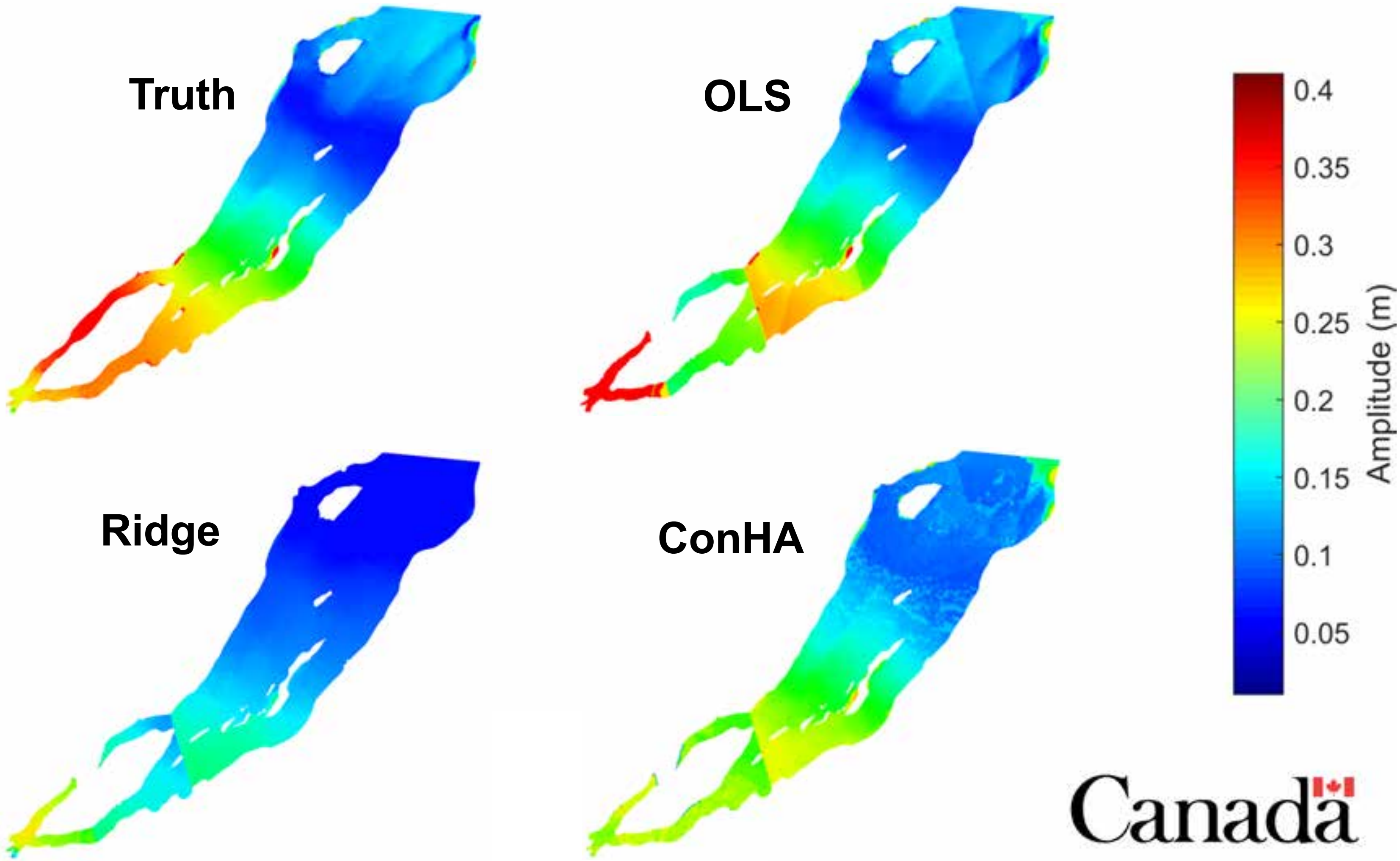
Mapping Estuarine Tides



- M_4 amplitude



Summary and Highlights

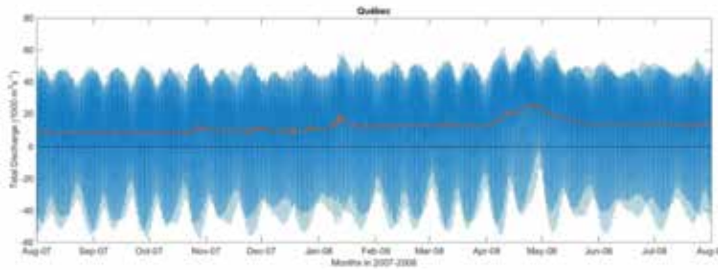
- Regularized least-squares
  - More robust than OLS, but sensitive to number of constituents
  - Longer record (~3 years) needed to improve tidal prediction
  - Good alternative if no in-situ data
- Constrained harmonic analysis (new approach)
  - Requires prior knowledge on tides at boundary stations
  - Robust to downsampling and reduced record length
  - Limited tidal aliasing
  - Resolves for multiple constituents in each tidal band
  - No prior spatial function needed (easily transposable)
  - Accuracy <8% during cal/val
  - Stable accuracy (<7%) achieved after 1 year of science mission

Perspective and Applications

- Future work
  - Full error analysis including SWOT errors
  - Non-stationary tides (e.g. influenced by river flow, storm surges)
    - NS_TIDE (Matte et al. 2013, *JTECH*)

- Extended uses
  - Coastal areas where tides are spatially coherent
  - Remote areas with limited field efforts
  - Model assimilation
  - Discharge estimation

Discharge Estimation



- Estimating river discharge from tides
  - Jay and Kukulka (2003), Moftakhari et al. (2013, 2016)

$$Q_R = \alpha + \beta \times TP^{\gamma}$$

  - Cai et al. (2014): analytical approach
- Tidal discharge estimation
  - Solving the continuity equation (Matte et al. 2018, *JGR*)

$$Q_{tot} = \sum_r Q_r - \int_{\Omega} \frac{\partial h}{\partial t} d\Omega$$

  - A new multi-gauge rating curve approach

$$Q = a + (bh_1 + ch_2)^d + (eh_2 + fh_3)^g$$