River discharge estimations, hydrological – hydraulic multi-fidelity models based on SWOT(-like) and multi-source data

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DassHydro open source softwares on GitHub : https://github.com/orgs/DassHydro-dev
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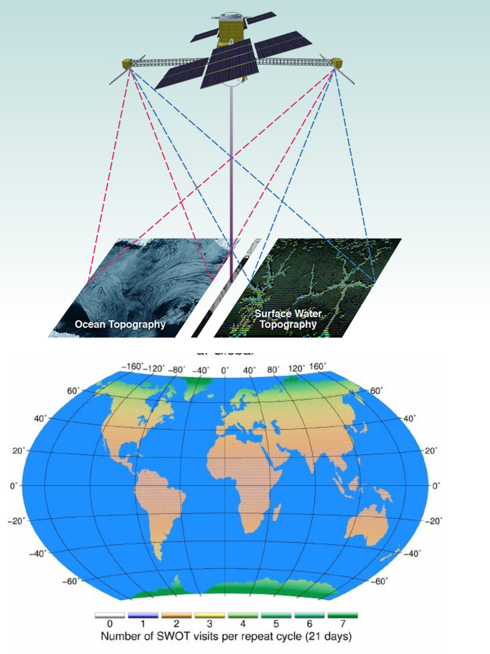
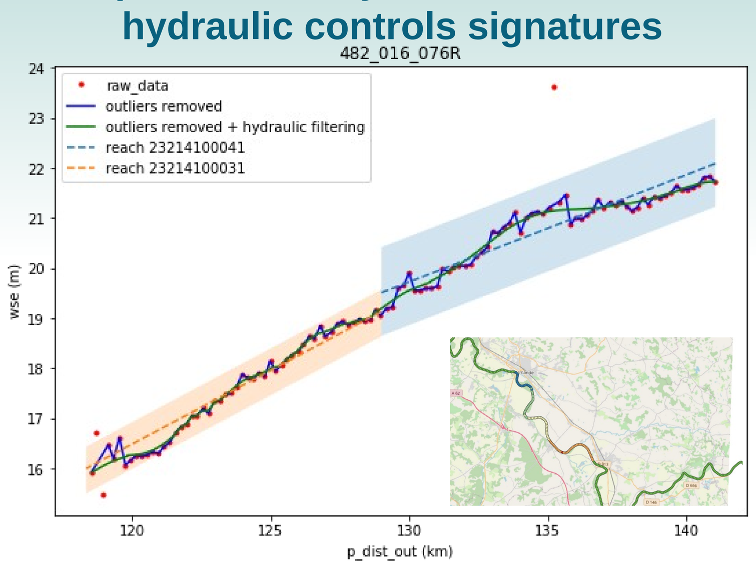
Context: worldwide hydraulic visibility of rivers surfaces variabilities with SWOT



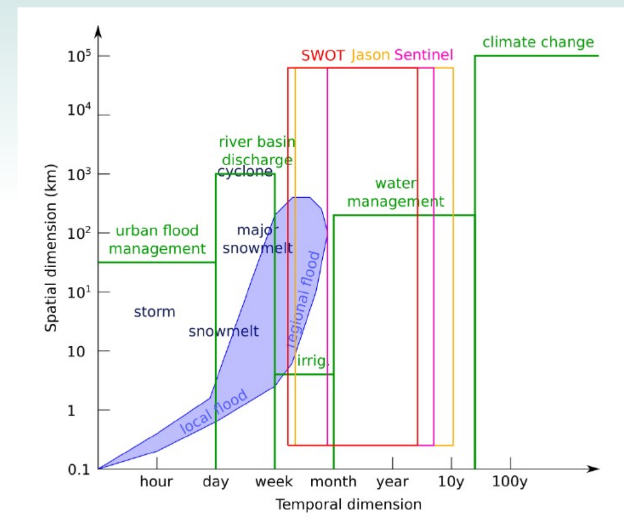
Number of SWOT revisits per 21d cycle. Biancamaria et al. (2016)

Spatial visibility of flow lines and hydraulic controls signatures



Example of 1D SWOT data on the Garonne River, spring 2023, hydraulic filtering (Larnier et al. 2023) flwg Montazem et al. (2019)

Visibility of hydrological processes signatures through networks



Spatio-temporal scales of visibility of hydrological processes (Pujol 2020, folwg Uhlemann [2013])

Challenges in Hydrology from WS signatures (SWOT, multi-mission altimetry, optical/radar water extents) :

- Estimation of rivers discharge (plus unobserved and uncertain bathymetry-friction) from water surface observables is an Ill-posed hydraulic inverse problem – local (at a section) plus spatial equifinality (Garambois-Monnier 2015, Larnier-Monnier-Garambois 2020, Garambois et al. 2020)
- Uncertainty (quantification) reduction by data assimilation of WS obs. (w/wo in situ data) in river networks-floodplains hydraulic models, feedback to hydrological model is faced with data uncertainties, heterogenity in nature and spatio-temporal sampling – SWOT temporal sparsity wrt higher hydrological frequencies (Brisset et al. 2016-18, Pujol et al. 2020)
- Regionalization of models parameters and learning of physical laws from massive data from reach to global scale

The HiVDI discharge algorithm

HiVDI = Hierarchical Variational Discharge Identification

- **Goal :** Estimating discharge of unmonitored rivers (plus effective bathymetry-friction) from the SWOT measurements plus global databases (SWORD, WBM).
- **Methodology** (Larnier et al. (2020); Larnier and Monnier 2023): hybrid physics-informed ML algorithm based on hierarchical flows models (shallow-water and Low-Froude systems), deep learning, variational data assimilation and Bayesian estimations
- Algorithm based on the **DassHydro open-source software**.

Hierarchical flow models

- Saint-Venant's equations (1D shallow-water)

$$\begin{cases} \partial_t A + \partial_x Q & = 0 \\ \partial_t Q + \partial_x \left(\frac{Q^2}{A}\right) + gA\partial_x Z & = -gA\, S_f \end{cases}$$

with $S_f \equiv S_f(A, Q; K) = \frac{|Q|Q}{K^2 A^2 R_h^{4/3}}$. B.C. : $Q_{in}(t)$ at inflow, normal depth at outflow.
Strickler K is reach (r) dependent : $K_r(h) = \alpha_r\, h^{\beta_r}$.

- Dedicated algebraic systems : low-complexity models Steady-state, low Froude assumption

$$D_c \cdot (\tilde{K}_{r,p} A_{r,0})_{R\times P} \; + \; D_d \cdot \tilde{K}_{RP} \; = \; \tilde{Q}_{RP}$$

with : $\tilde{K}_{RP} = (K_{r,p}^{3/5})_{r,p} \in R^{RP},\; A = (A_{r,0})_{r,} \in R^R$
$\tilde{Q}_{RP} = (Q_{r,p}^{3/5})_{r,p} \in R^{RP}$.

- River description from SWOT measurements.

R reaches ($\approx$ 200 m long, RiverObs), P overpasses.

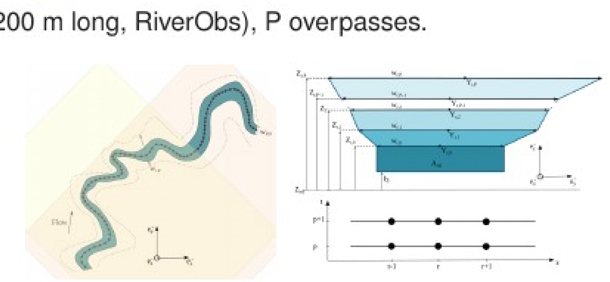


FIGURE – (Right Top) Effective river cross section at reach r defined from SWOT data set $\{Z_{r,p}, W_{r,p}\}_{R,P+1}$. (Right Bottom) Space - time stencil (r, p). x denotes the curvilinear abscissa along the river center line defined at low flow by $Y_{r,0}$ with $Y_{r,p}$ the middle of the cross sectional width.

Mathematical ingredients

- **Variational Data Assimilation**
c vector of the unknown "parameters" :

$$c = (\{Q_{in}\}_{1..P};\ \{b\}_{1..R};\ \{\alpha, \beta\}_{1..R}) \in \mathbb{R}^{P+2R}$$

Optimisation :

$$\min_k J(k) \text{ with } k = B^{-1/2}(c - c_{prior})$$

$$J(k) = j(c) \;=\; \|Z(c) - Z^{obs}\|_N^2 \;+\; \gamma_{reg}\, j_{reg}(c)$$

$Z(c)$: WSE, output of the flow model.
$B = diag(B_Q, B_b, B_K)$ covariance matrices (probabilistic).
Gradient of $J(k)$ computed from the *adjoint model* obtained by Algorithmic Differentiation of the direct code (DassFlow software).

- **Deep learning**
The training dataset D contains learning pairs (samples) (I_i, Q_i), $i = 1, \cdots, N_{lp}$. The i-th input is $I_i = (dA, W, S, \mathcal{A})_i$, the i -th values at the considered location and day.
dA (m^2) : variations of the wetted crossed-sections above the unobserved A_0. $\mathcal{A}$ (km^2) : extracted from HydroSHEDS database
Training an ANN consists to solve : $W^* = arg\min_W I_Q(W)$ with the loss function (misfit-cost function) I_Q set as

$$I_Q(W) = \|Q(W) - Q^{obs}(I)\|^2_{2,N_{ls}} \quad (1$$

The resulting estimator is : $Q^{(ANN)} = Q(W^*; I)$.

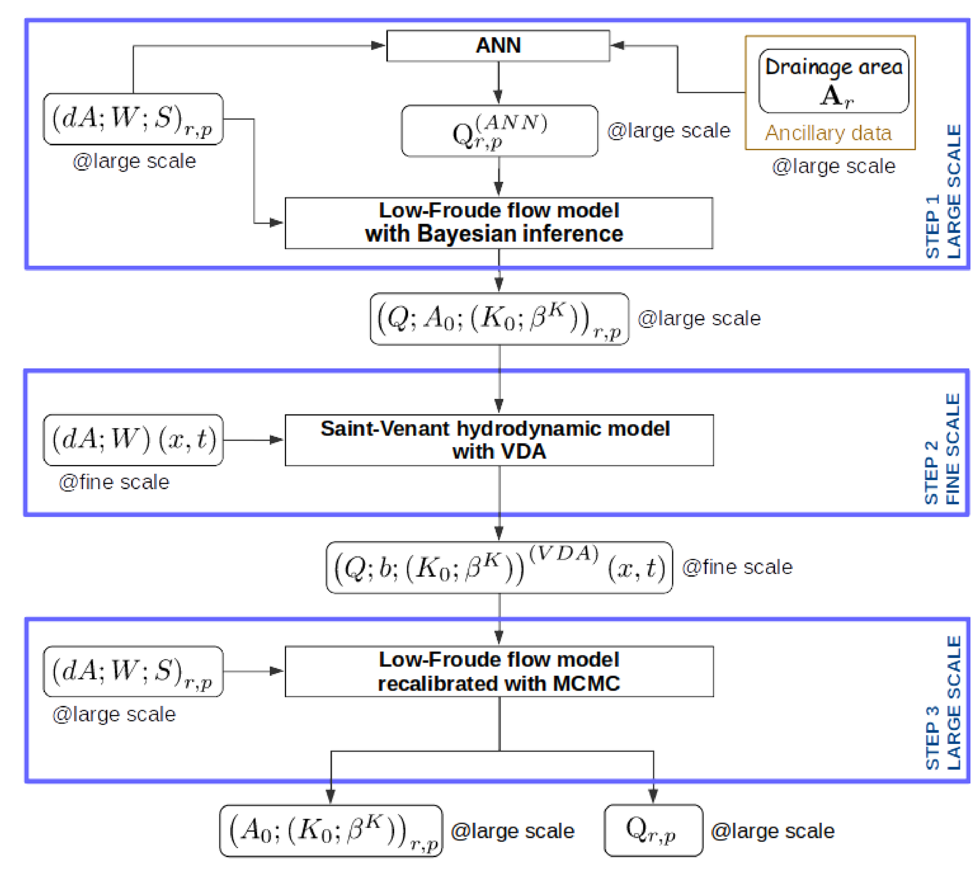
- **Bayesian estimations**

$$p(k_\square | Z^{obs}) \propto \text{ likelihood } p(Z_{obs}|k_\square) \times \text{ prior } p(k_\square)$$

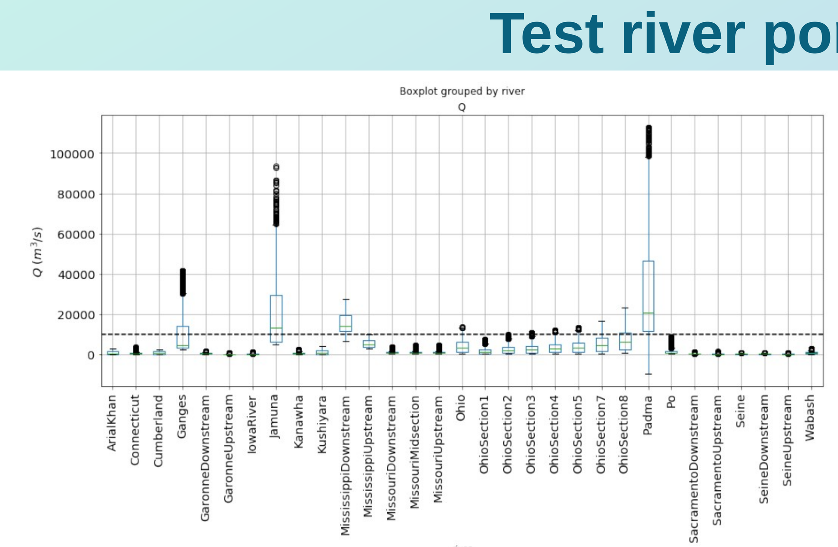
for $k_\square$ = discharge or bathymetry.

HiVDI Flowchart

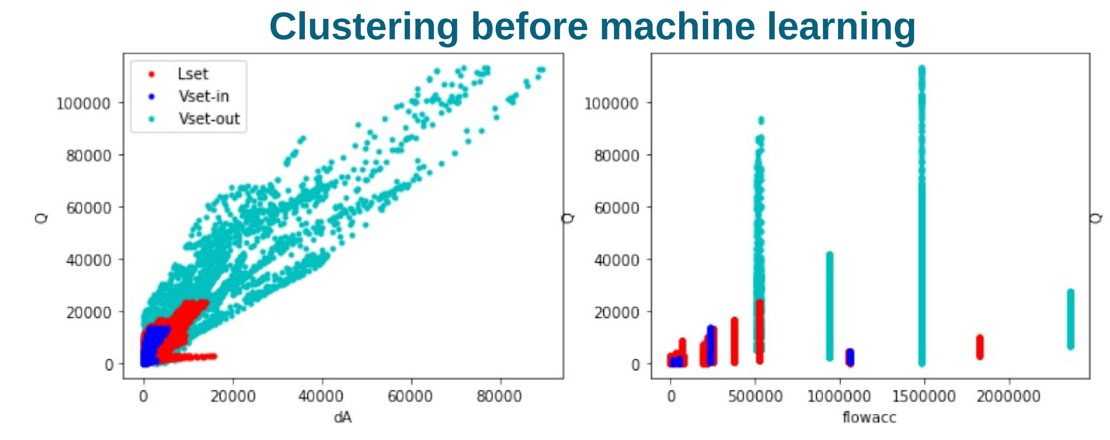
Estimation of rivers discharge in two stages :
1) "Learning phase" (1 year). Calibration of the models VDA / Deep Learning / Bayesian estimations.
2) Real-time estimations given new measurements. Low-complexity flow model.



Test river portions (« PEPSI » datasets)

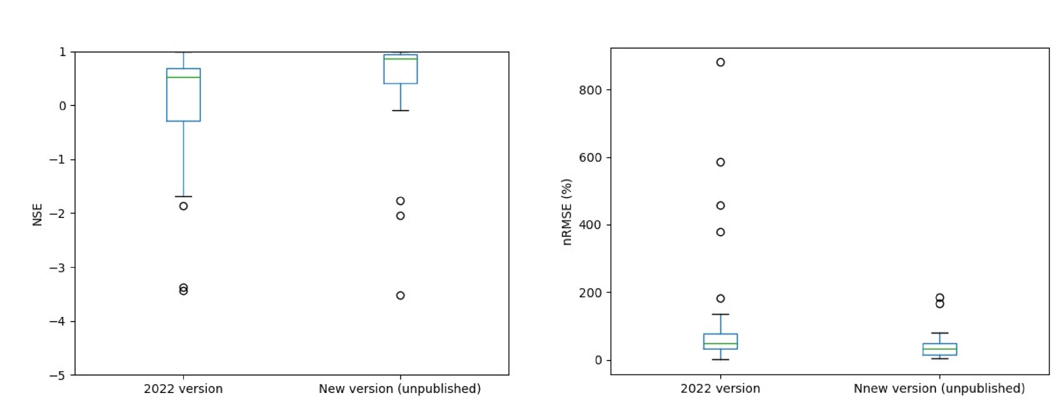


Clustering before machine learning

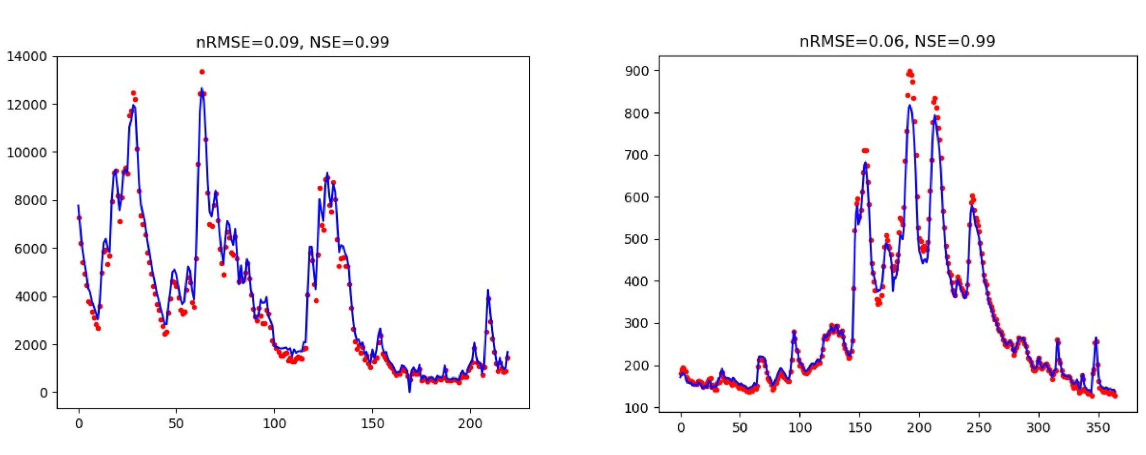


Numerical results

HiVDI Inversions on 45 River portions

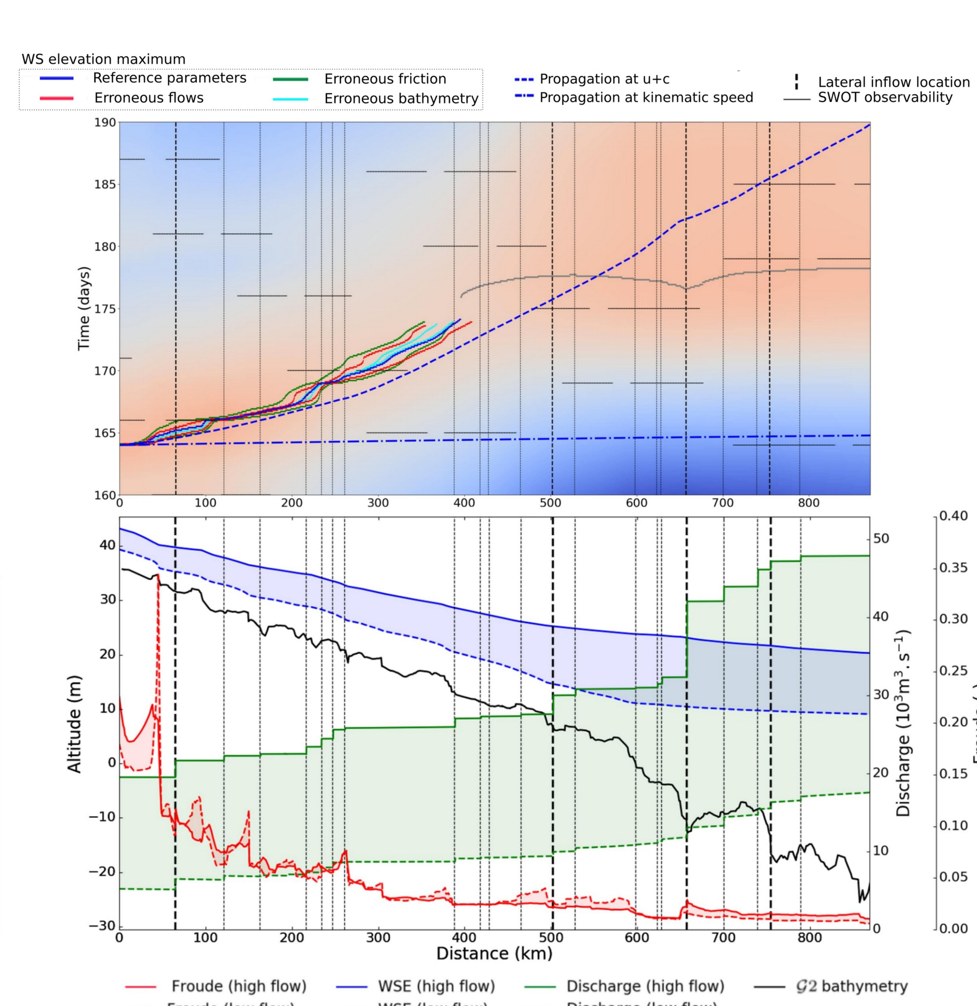


Overall performance of previous and new version of HiVDI (including ML-Bayesian determination of prior for VDA) on 45 river portions worldwide,

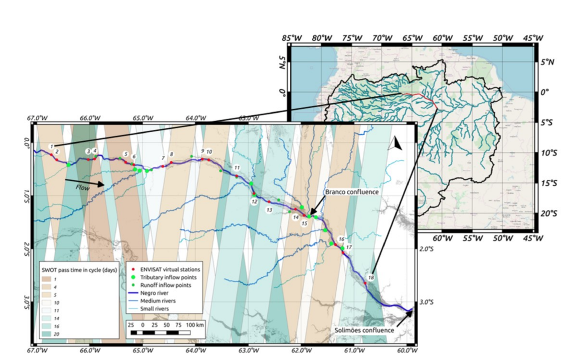


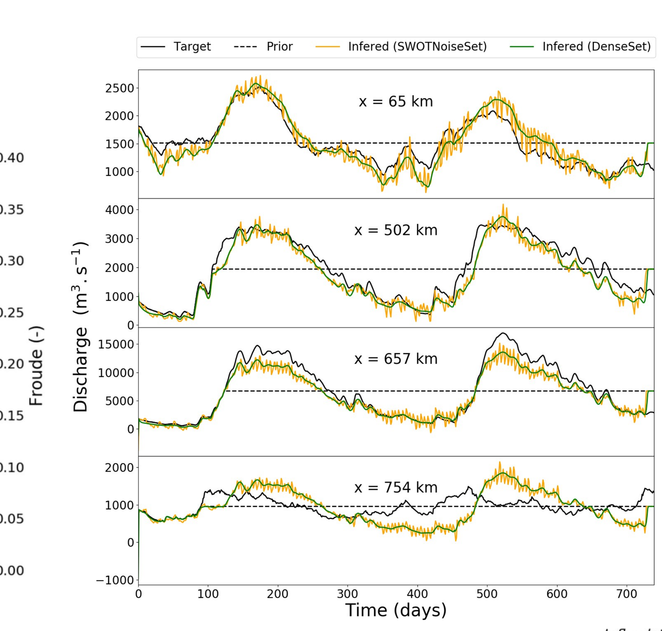
Exemple of discharge Q(t) estimated on (Left) OhioSection5, (Right) Seine.

Hydrological-Hydraulic model based on multi-source data



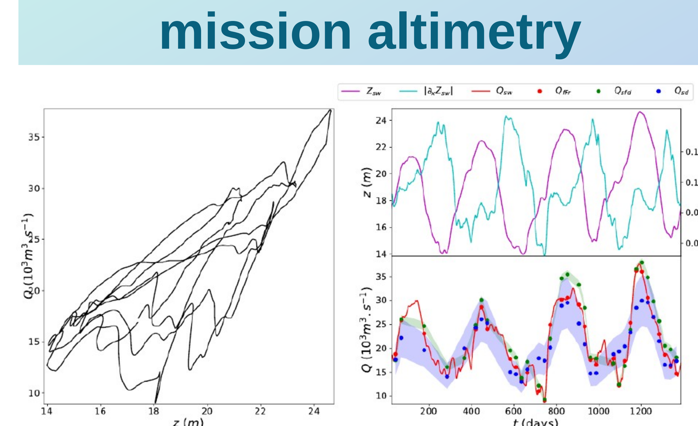
(top) Identifiability map from hydrological-hydraulic model of 870km of the Negro River, based on nadir altimetry and water mask ; SWOT swathes in grey. (bottom) 1D river model view (Pujol et al. JOH 2020)





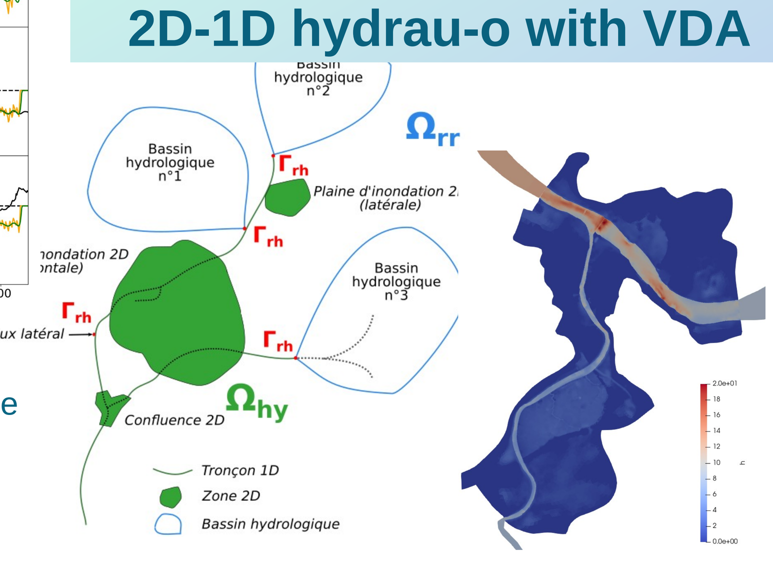
Infered discharge hydrographs at multiple inflow points from swot like data

Model derived Stage-Fall-Discarge laws for multi-mission altimetry



Z-Q hysteresis, backwater affected reach o the Negro River, SFD estimation of discharge with uncertainty from **elevation-slope** (Malou et al. JOH 2020) $Q = \alpha W^{\beta}(Z - b_{sfd})^{\gamma} S^{\delta} + \epsilon_Q$

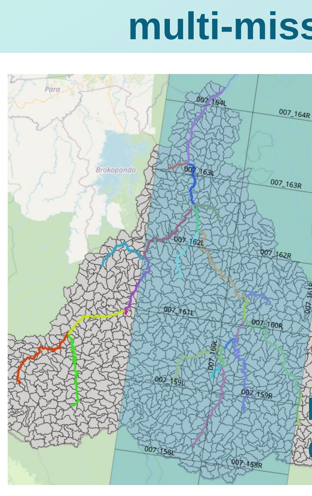
2D-1D hydrau-o with VDA



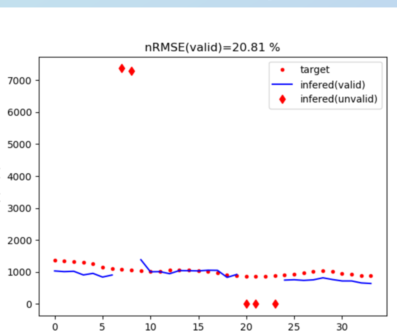
Hydraulic Information feedback to hydrological mode parameters calibration (Pujol et al. GMD, 2022)

Towards HiVDI application to full rivers networks

multi-mission altimetry + radar water masks ; SWOT



Maroni hydro-au model, Icesat Z(x,t) and sentinel derived water mask W(t)



HiVDI with real SWOT data, Maripasoula, Maroni

Ongoing work and perspectives – River Networks & Deep Learning

- Effective open source algos – operational implementation, deep learning from SWOT at rivers networks, regional worlwide scales, with other databases and hybrid hydrological regionalization (cf. LSTM in Hashemi et al.,HESS 2022, hybrid regio HDA-PR in Huynh et al. ARXIV, 2023)
- Research of Improvements - Hybrid approaches ML-VDA, hydo-au regionalization from SWOT and multi-source DA, UQ, multiresolution 2D-1D hydrau-o [Pujol et al.,GMD 2022]

Main references related to discharge inference with SWOT, models. On equifinality issues : [Garambois-Monnier, AWR'15], [Larnier-Monnier-Garambois et al., IPSE'20]. On identifiability capabilities : [Brisset-Monnier-Garambois et al., AWR'18]. On accurate flow solvers : [Monnier-Couderc et al., AWR'16]. On algorithm evaluations : [Tuozzolo et al. GRL'19], [Frasson et al., WRR'21]. Latest version of HiVDI and codes : [Larnier-Monnier, Comput. GeoSc.'23], [Pujol et al., GMD'22]. Model learnt Stage-Fall-Discharge laws : [Malou et al. JOH 2022]. Physical kernels for VDA: [Malou-Monnier, InvPB, 2022]. On complex flows investigations - applications :[Garambois et al. Hydro. Proc'17], [Garambois et al. JoH'20], [Pujol et al., JoH'20]. On SWOT data segmentation-filtering : [Montazem et al., GRL'19]